Flowers and Steps in the Boolean Lattice of Hexagrams*

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Overview

This paper compares some techniques introduced by three contemporary authors on the Yi Jing and shows how each of their individual methods can be seen as a special case within a general structural theory of the Yi Jing. The main ideas that I compare are Edward Hacker's *Hexagram Flowers*, Mondo Secter's *Hexagrams of Change* and *Transition Hexagrams*, and Stephen Karcher's *Steps of Change*.

I begin by outlining the ideas of each author, concentrating mainly on the structural aspects of their techniques. I then provide some initial comparisons between the different ideas, showing their common themes. The next section introduces some necessary background in lattice theory as applied to the Yi Jing, including a brief mention of some historical context for the approach. The formal content of this presentation is kept to a minimum; readers interested in the details of the mathematics should refer to my earlier work on Boolean Algebra and the Yi Jing.[1] I then show how each of the author's ideas presented can be located as special cases in the Boolean lattice of hexagrams.

Three Contemporary Thinkers

There are many contemporary authors working on the Yi Jing in the English language, with a wide variety of new ideas being presented. The goal of this paper is to show how apparently different approaches to the structure of Change can be reconciled within a single framework. I begin by providing a brief description the ideas of each author.

Edward Hacker

Hacker's *I Ching Handbook* presents a technique called *Hexagram Flowers*.[2] This works as follows: starting from a centre hexagram, change each line in turn to its opposite to generate a new hexagram. Each new hexagram is called a petal. All the lines are changed, so each flower centre always has six petals, thereby enumerating all hexagrams that are *minimally* different to the centre hexagram (*i.e.*, different in a single line).

For example, if we start with the hexagram 60 *Limitation*[3] as the centre then, starting at the bottom and changing each line in turn, we get the following six petals: 29 *The Abysmal*, 3 *Difficulty at the Beginning*, 5 *Waiting*, 58 *The Joyous*, 19 *Approach* and 61 *Inner Truth*. This is shown in Figure 1 below.

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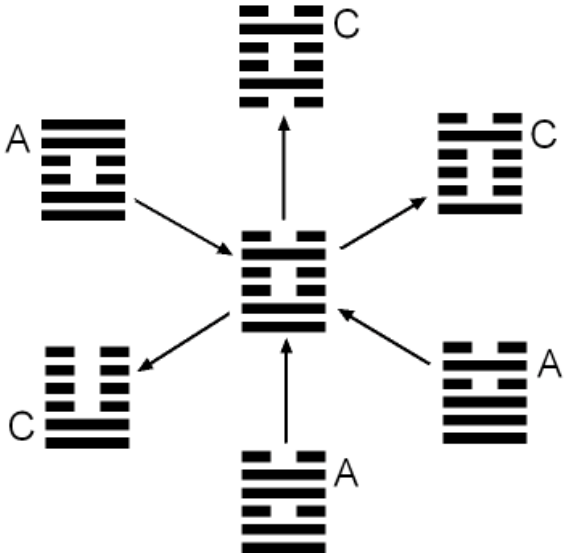

Figure 1: Hexagram Flower for 60 *Limitation*

Hacker describes a Hexagram Flower as defining a set of causal relationships between the situations described by the hexagrams. Petals with an additional yang line are taken to be *antecedent* to the centre whilst petals with an additional yin line are taken to be *consequent* to the centre. This is derived from the probabilities of the traditional yarrow stalk divination method, where a yang line changing to yin is three times more likely than a yin line changing to yang. In Figure 1 these causal relationships are conveyed by the direction of the arrows: the arrow goes from the antecedent to the consequent. So, 5 *Waiting*, 58 *The Joyous* and 61 *Inner Truth* are considered to be likely antecedents to 60 *Limitation*; whilst 29 *The Abysmal*, 3 *Difficulty at the Beginning* and 19 *Approach* are taken to be its likely consequents.

As an aside, this interpretation of the relative amounts of yin and yang in terms of causal relationships means that hexagrams with more yang lines have fewer antecedents and more consequents, whilst hexagrams with more yin lines have fewer consequents and more antecedents. Taking this to the extreme point means that *The Creative* has no antecedents, it is itself the universal antecedent. Further, *The Receptive* has no consequents, it is itself the universal consequent. Hacker does not explore the ramifications of this view.

Mondo Secter

Secter's book, now also called *The I Ching Handbook*, contains a number of novel ideas.[4] In this paper I am concerned with two techniques in particular: the idea of *Change Hexagrams* and the method of *Transition Hexagrams*.

As part of his "decision-making without divination" technique (pp158–176) Secter introduces Change Hexagrams. Within the context of using the constituent trigrams to represent two parties in a single situation, then "the six lines of the hexagram indicate the six directions or paths the relationship or situation can take" (p163). The technique involves changing one of the lines of the hexagram to its opposite and there are therefore six possible Change Hexagrams for any given initial hexagram.

Clearly, this technique is formally the same as Hacker's Hexagram Flowers. However, there is no suggestion of a causal relationship between the figures. Instead, Secter

suggests that the Change Hexagrams can be used to chose the best direction from a particular situation. For example, if the current situation was symbolized by 60 *Limitation*, with the lower trigram representing my role in the relationship and the upper trigram representing my partner's role, then the hexagrams 29 *The Abysmal*, 3 *Difficulty at the Beginning*, and 5 *Waiting*, would represent the paths initiated by my changing, whilst the hexagrams 58 *The Joyous*, 19 *Approach* and 61 *Inner Truth* would represent possible paths resulting from my partner changing.

As part of this technique, Secter provides a table of trigram relationships (p165). The table shows, for those trigrams with two common lines, which line differs. I reproduce this, in a slightly different format to Secter, as Table 1.

		B	M		T			
	B			M		T		
	M			B			T	
		M	B					T
	T					B	M	
		T			B			M
			T		M			B
				T		M	B	

Table 1: Secter's Trigram Differences

The intended use for this table, is to help select which trigrams are candidates for change when determining the Change Hexagrams. Thus, in 60 *Limitation*, the bottom trigram is *Lake*; by reading across the relevant row in the table we can see that changing the middle line (M) would give us *Thunder*, changing the bottom line (B) would give us *Water*, and changing the top line (T) would give us *Heaven*. The relationships that this table encodes will be explored when we introduce the lattice technique below.

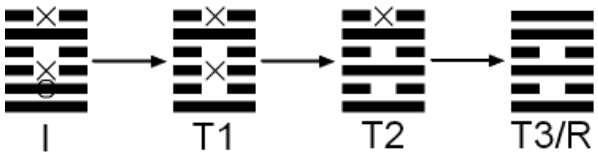

Figure 2: Transition Hexagrams for 796876

Secter also introduces the notion of Transition Hexagrams (pp64–69). These are used to describe the individual steps of the transition in a hexagram with changing lines. They are derived by allowing each changing line in the figure to change in sequence, from the lowest to the highest; each changing line gives a new hexagram that serves as the base for the next change. For example, in Figure 2, we start with 60 *Limitation* as

the Initial hexagram, with changing lines in the second, third and top positions. Starting from the bottom, the first line to change is the nine in the second place, which gives a new hexagram of 3 *Difficulty at the Beginning*. This is the first Transition Hexagram, marked as T1 in the diagram. The next line to change is the six in the third place of 3 *Difficulty at the Beginning*, which gives the second Transition Hexagram 63 *After Completion*, marked as T2. The final changing line is the six at the top of 63 *After Completion*, which gives the final hexagram, T3, of 37 *The Family*. Note that the final Transition Hexagram is also the Resulting hexagram, which is generated by changing all of the lines in the Initial hexagram.

Secter presents this technique as a way of dealing with the complexities that arise when there are multiple changing lines in a reading. His idea is that the first changing line causes a "fundamental modification" to the Initial hexagram, which alters the context for subsequent changing lines. Thus, after the nine in the second place changes in 60 *Limitation*, the context for the next changing line is 3 *Difficulty at the Beginning*. This process is repeated until all the changing lines have been handled and the Resulting hexagram is reached. So, if there are *n* changing lines in the Initial hexagram, there will be *n* Transition Hexagrams, with the final hexagram being the Resulting hexagram. Including the Initial hexagram, this makes for a total of *n+1* hexagrams in a reading with *n* changing lines. The final Transition Hexagram is *always* the Resulting hexagram.

Stephen Karcher

Karcher has developed the ideas in his *Tools for Change* material over a number of publications.[5] Tools for Change is a collection of techniques that are used to expand on the material available for interpretation in a reading. In this paper I am interested in one particular technique from the collection, called the *Steps of Change*. Like Secter's Transition Hexagrams, the Steps of Change are intended to provide additional information to help interpret multiple changing lines in a reading. However, the manner of their generation is slightly different, giving a different set of hexagrams to consider.

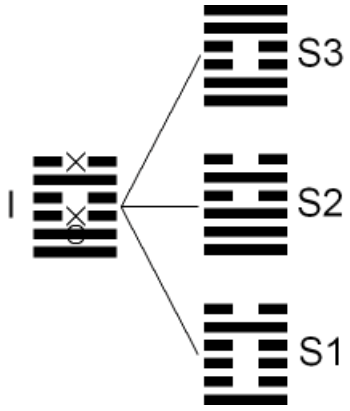

Figure 3: Steps of Change for 796876

Karcher's Steps of Change are generated by taking each changing line in turn and allowing it to change, in isolation, relative to the Initial hexagram. An example of this is shown in Figure 3 for the same reading explored in Figure 2, 60 *Limitation* with changing lines in the second, third and top places.

In this example, changing the nine in the second place of 60 *Limitation* gives the first step of 3 *Difficulty at the Beginning*. Then, changing the six in the third place of 60 *Limitation* gives 5 *Waiting*, and finally, changing the six at the top of 60 *Limitation* gives 61 *Inner Truth*. Thus, each line change is done with respect the original Initial hexagram, rather than allowing each line change to act cumulatively through the process.

Notice, in this case, that none of the Steps of Change is the same as the Resulting hexagram. In the case of a reading with *n* changing lines, there are *n* Steps of Change. If there is only one changing line, then the single Step of Change is the same as the Resulting hexagram. If there are multiple changing lines, then none of the Steps of Change is the same as the Resulting hexagram.

Initial Comparisons

I have considered a range of contemporary techniques for extending the traditional analyses of hexagram interpretation. In the final part of this article I shall show how all of these techniques can be placed within the unifying framework of Boolean Lattice theory applied to the Yi Jing. However, I shall first present some initial comparisons between the different ideas on their own terms.

Hexagram Flowers

It is clear from their descriptions that Hacker's Hexagram Flowers and Secter's Change Hexagrams use the same formal mechanism: for any particular starting hexagram both of them provide an enumeration of those hexagrams that differ in a single line. However, the two authors provide different interpretations for the technique. For Hacker, there is a probabilistic causal relationship between the petal hexagrams and the centre. Depending on whether a yin or a yang line must be changed to get to the centre, the petal is either a consequent or an antecedent of the centre. For Secter, the related hexagrams provide a way of exploring the likely effects of making any particular change in the situation and there is no causal bias to the changing lines. Which of these two interpretations might be preferred will be considered when we look how the basic formal mechanism of Hexagram Flowers fits in to the lattice theory.

It is also interesting to note that the hexagrams generated by Karcher's Steps of Change technique will always be selected from the petals of the Initial hexagram's flower. Technically, the Steps of Change hexagrams form a subset of the Hexagram Flower petal hexagrams. Which petals from the flower are selected is determined by which lines are changing in the initial hexagram.

Steps and Transitions

Secter's Transition Hexagrams and Karcher's Steps of Change are both presented as techniques for describing the process of change from one situation to another. Both handle multiple changing lines by generating a new hexagram for each changing line in the Initial hexagram. Secter's technique does this cumulatively, stepping from the Initial hexagram through a sequence of Transition Hexagrams to the Resulting hexagram. Each step in the sequence is related to the previous step. Karcher's technique does not provide a sequence in the same way, but instead produces a set of hexagrams, each related to the Initial hexagram, rather than to the previous step.

When only one line changes, for Secter, there is one Transition Hexagram, which is identical to the Resulting hexagram. In the same situation, for Karcher, there is a single Step of Change, which is also identical with the Resulting hexagram. When more than one line changes, for Secter, there will be a Transition Hexagram for each changing line, with the final Transition Hexagram being the Resulting hexagram. For Karcher, there are also as many Steps of Change as there are changing lines; however, when there are multiple changing lines, none of the Steps of Change are the same as the Resulting hexagram. We shall see later that the Boolean lattice gives us a way of explaining these differences.

Basic Lattice Theory

Before exploring the relationships between these different techniques in more detail, we need to consider some basic lattice theory.[6] In application to the Yi Jing, a Boolean lattice gives us a way of arranging the figures, which represents both the relative amounts of, and the structural distribution of, yin and yang.

Complete Lattice Structures

We shall start with the simplest structure from the Yi Jing, a single line. The lattice for this is shown in Figure 4.

Figure 4: The Line Lattice

This structure gives us the basic orientation for all further structures. Yang is shown as the top of the lattice and yin as the bottom: in the words of the *Shuo Gua*, "heaven and earth determine the direction".[7] Thus, the lattice provides as with a way of ordering the figures. Now consider Figure 5, the lattice for bigrams.

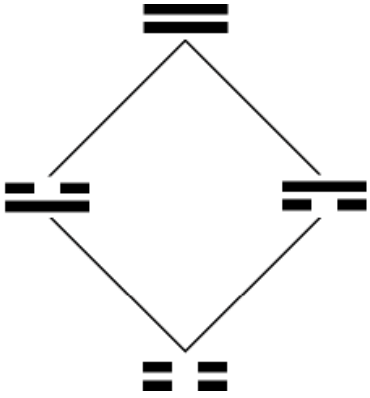

Figure 5: The Bigram Lattice

As with the lattice for single lines, pure yang is the top of the lattice and pure yin is the bottom. Between them are the two bigrams with one line each of yin and yang. This shows the beginnings of how the lattice orders the figures according to the amount of

yin and yang that they contain: the lowest layer has no yang, the middle layer has one yang line and the top layer has two yang lines.

The lattice for trigrams is shown in Figure 6. The additional complexity in this structure shows how the lattice also represents the structural distribution of yin and yang in a figure as well as simply the relative amount.

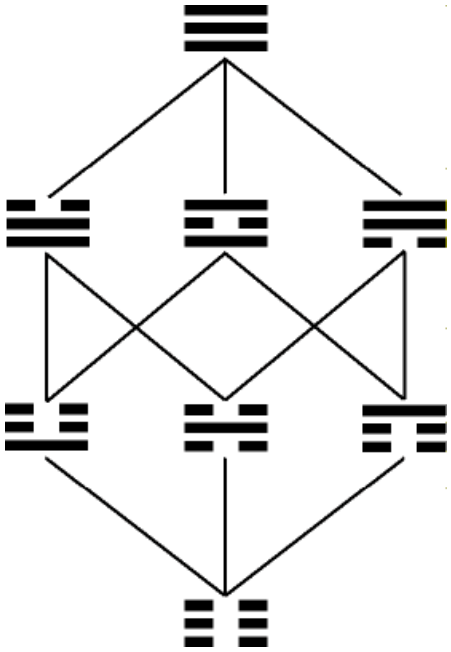

Figure 6: The Lattice for Trigrams

As before, pure yin (*i.e.*, no yang) is at the bottom and pure yang (*i.e.*, three yang) is at the top of the structure. Then, working up from the bottom, the second row contains all the trigrams with a single yang line, and the third row contains all the trigrams with two yang lines. So, as before, the lattice structure encodes the relative amounts of yin and yang in the figures. However, we can now see that it also encodes the structural distribution of yin and yang. Consider the trigram *Thunder* in the second row and notice that it is connected to *Lake* and *Fire* in the third row, but not to *Wind*. This is because *Lake* and *Fire* both have a yang line in a common position with *Thunder*, but *Wind* does not. Similarly, *Wind* in the third row is connected to *Mountain* and *Water* in the second row, but not to *Thunder*. This is because *Wind* has a yin line in a common position with *Mountain* and *Water*, but not with *Thunder*. Thus we see how the lattice structure encodes the structural aspects of the distribution of yin and yang in a figure as well as the amount.

The full Boolean lattice for the hexagrams is too complex to show here. However, it follows the same principle as the trigram lattice. The bottom row contains the single hexagram *The Receptive* and the top row contains the single hexagram *The Creative*. The second row contains the six hexagrams with a single yang line, the third row contains the fifteen hexagrams with two yang lines, the fourth row contains the twenty hexagrams with three yang lines, the fifth row contains the fifteen hexagrams with four yang lines, and the sixth row contains the six hexagrams with five yang lines. Finally, the hexagrams in each row are connected to those hexagrams in the immediately adjacent rows, which differ by a single line, showing the structural distribution of yin and yang in the hexagrams.

Lattice Locales and Sublattices

As well as looking at a complete lattice structure it is also important to be able to isolate particular parts of a lattice for study. There are two techniques that I shall present here: the idea of a lattice locale for a figure, and the idea of a sublattice.

A key idea for understanding some of the work discussed above is what I shall refer to as the *lattice locale* of a figure. Figure 7 shows the lattice locale for the trigram *Fire*. Compare this to Figure 6 to see exactly what part of the lattice is being considered. The lattice locale for any particular figure is composed of the adjacent figures in the lattice to which it is directly connected. Because of the structural properties of the lattice this means that the lattice locale of any particular figure will contain all those figures that differ from it by a single line.

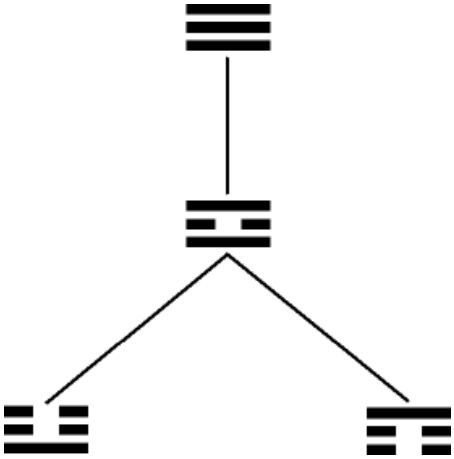

Figure 7: The Lattice Locale for Fire

The final idea from lattice theory that I need to introduce here is the idea of a *sublattice*. Like a lattice locale, a sublattice provides us with a way of looking at part of the total lattice structure by focussing on the particular elements of a figure in which we are interested.

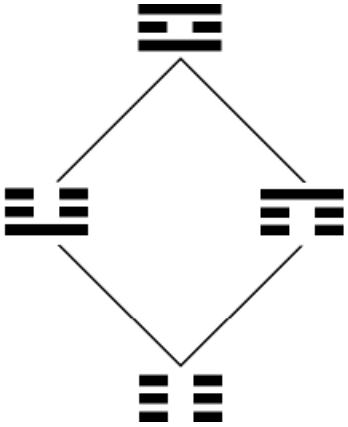

Figure 8: A Sublattice for Fire

For example, if we are only interested in what happens when the top and bottom lines of the trigram *Fire* change whilst the middle line remains constant, then the result will be the sublattice shown in Figure 8. Again, comparison with Figure 6 will clarify exactly what part of the full lattice is being considered.

Note how, when only two lines of a more complex figure are allowed to change, the resulting sublattice has the same shape as the bigram lattice shown in Figure 5. This would also be the case if we were looking at a sublattice for a hexagram where only two lines were allowed to change. Similarly, if we were to take a hexagram and allow only three of the lines to change, then we would get a sublattice that was the same shape as the trigram lattice in Figure 6. We shall return to this idea below.

Historical Context

It is interesting to note, at this point, that Chinese research into the Yi Jing had previously made some steps towards the use of the Boolean lattice. The compendium *Zhouyi Tuishi Dadian* contains a vast collection of diagrams related to the Yi Jing.[8] For example, on page 661 there is a diagram that bears a striking similarity to the Boolean lattice for hexagrams: the hexagrams are arranged in rows, with the bottom row containing only the hexagram *The Receptive* and the top row containing only the hexagram *The Creative*; in between, each row contains the hexagrams with one, two, three, four and five yang lines respectively. The only thing missing that the Boolean lattice contains, is the interconnecting lines that encode the structural distribution.

In his book *I Ching Mandalas*,[9] Cleary shows a similar diagram to the one in *Zhouyi Tuishi Dadian*. He describes this as "The Six Yangs and Six Yins of the Yang Fire and Yin Convergence" and says that this diagram is "used to study yang and yin in specific positions and to study yang and yin in specific concentrations" (p39). Cleary's characterisation of this diagram is exactly what the Boolean lattice of hexagrams encodes. Thus, we might conjecture that, irrespective of the level of their mathematical sophistication, Chinese scholars of the Yi Jing had an intuitive grasp of some of the lattice properties of the symbols of Change.

Lattice Explanations

We shall now show how the lattice theory outlined above allows us to provide a unifying framework for the different ideas from the authors under discussion.

Lattice Locale

I noted above that Hacker's Hexagram Flowers and Secter's Change Hexagrams were formally identical. In point of fact, both techniques are using the lattice locale of a hexagram to provide an interpretative mechanism.

Figure 9 shows the lattice locale for 60 *Limitation*; compare this to Figure 1, the corresponding Hexagram Flower. Clearly, the same hexagrams are involved in both cases. Further, because of the ordering of the relative amounts of yin and yang encoded by the lattice, all the antecedent petals from the flower appear in the top layer of the lattice locale whilst all of the consequent petals appear in the lower layer of the locale. Thus, the lattice locale provides a natural categorization of the petals according to Hacker's causal criterion. However, this should not be taken as automatic confirmation of Hacker's interpretation. What the lattice encodes is the relative concentration of yin and yang. This only gives rise to a causal bias once the idea of yarrow probabilities are added to the description.

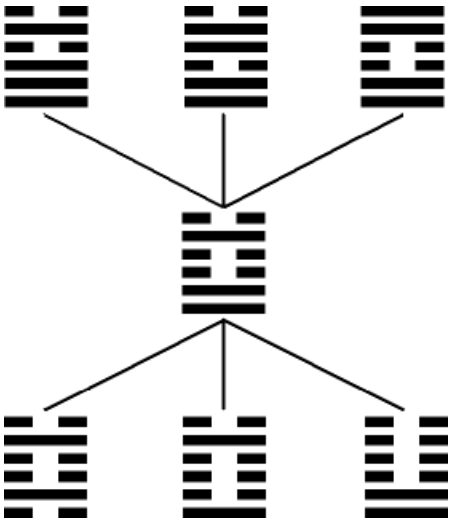

Figure 9: The Lattice Locale for 60 *Limitation*

We can also look at Secter's table of matching trigram lines (see Table 1) in the context of the lattice locale. If we compare the entries in the table for the trigram *Fire* with the lattice locale for the same trigram shown in Figure 7, then we see that those trigrams which the tables picks out are exactly those trigrams which form the lattice locale. This is not surprising given that the table reflects, at the trigram level, what is going on at the hexagram level in the Change Hexagrams. What this shows is that a lattice locale for a particular hexagram can be decomposed into the two lattice locales for its upper and lower trigrams.

The Sublattice of Change

In order to compare Secter's Transition Hexagrams with Karcher's Steps of Change it is necessary to see how the changing lines in a particular hexagram generates a sublattice. To show how this technique develops I shall consider three cases: each time starting from 60 *Limitation* I shall consider a single changing line, two changing lines and three changing lines. For each example, I shall show how the resulting sublattice provides a formal context for unifying Secter's and Karcher's separate techniques.

First, consider the case where the Initial hexagram 60 *Limitation* has a nine in the second place giving a Resulting hexagram of 3 *Difficulty at the Beginning*. This gives rise to the very simple sublattice shown in Figure 10.[10]

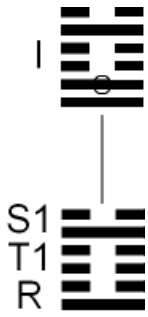

Figure 10: The Sublattice for 798878

Hexagram 3, *Difficulty at the Beginning* is one of the petals of the hexagram flower for 60 *Limitation*, as expected from our earlier discussion. In this case, it is also the

Resulting hexagram from the changing line. For Secter, it is the single Transition Hexagram and for Karcher, it is the single Step of Change. Thus, in the most basic case, the sublattice analysis captures both the Secter and Karcher analyses.

We shall now consider the case of two changing lines: 60 *Limitiation*, with a nine in the second place and six in the third place. This will give a Resulting hexagram of 63 *After Completion*. The sublattice for this is shown in Figure 11. Remember that a Boolean lattice for hexagrams is always arranged so that the hexagrams with most yang lines are towards the top of the diagram: 60 *Limitation*, the Initial hexagram, is at the left of the diagram, with 3 *Difficulty at the Beginning*, the result of changing only the nine in the second place, at the bottom of the diagram. 5 *Waiting*, the result of changing only the six in the third place is at the top of the diagram, and finally, 63 *Before Completion*, the result of changing both lines, is at the right of the diagram. To help visualise the flow of change through the sublattice, I have also added arrow heads to the lattice lines.

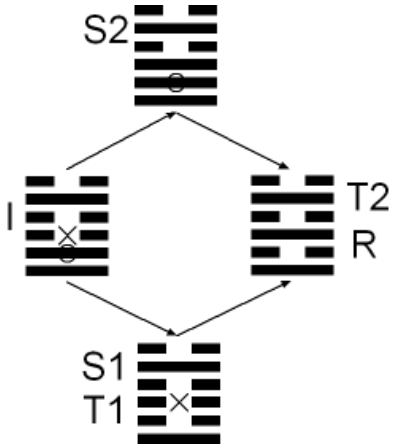

Figure 11: The Sublattice for 796878

In this case, Secter's Transition Hexagram analysis consists of the left, bottom and right hexagrams from the diagram: this is a path from I to R through the sublattice. On the other hand, Karcher's two Steps of Change would be the top and bottom hexagrams: these are the hexagrams immediately adjacent to I in the sublattice. So, again we see that the sublattice analysis captures both the Secter and Karcher analyses.

We shall now return to consideration of the case with three changing lines. This will be 60 *Limitation*, with a nine in the second place, a six in the third and a six at the top. Secter's Transition Hexagram analysis of this was shown earlier in Figure 2 and Karcher's Steps of Change analysis in Figure 3. The corresponding sublattice for this change is shown in Figure 12.

Looking at this diagram it is clear that Secter's Transition Hexagrams trace one particular path through the sublattice from the Initial hexagram to the Resulting hexagram. It is not the only possible path; it is the path that results when the relevant lines of the Initial hexagram are changed from the bottom to the top in sequence. If the lines are allowed to change in some other order, then a different path will result. Conversely, Karcher's Steps of Change mark the starting point of each of the possible paths through the sublattice from the Initial hexagram to the Resulting hexagram. That is, Karcher's analysis represents the possible *first steps* that might be taken to start the process of transition from the Initial to the Resulting hexagram.

The sublattice of change provides a general map that contains all of the possible routes one might take from the Initial to the Resulting hexagram. When there are many changing lines, the number of possible routes becomes large. Secter's technique restricts us to considering only one route and therefore simplifies the problem. Karcher's technique simplifies the problem in a different way, by only considering the possible first steps. Whilst both of these simplifications are valuable, I believe that they restrict the range of possibilities. For example, if it is felt that a particular transition really starts in the outer trigram and then manifests within, Secter's path through the lattice will not do. Equally, Karcher's technique only shows us the possible first steps, and does not take us to the conclusion of the change. By considering both of these techniques in the context of the sublattice of change, we can greatly expand on the range of images available for interpretation in a reading.

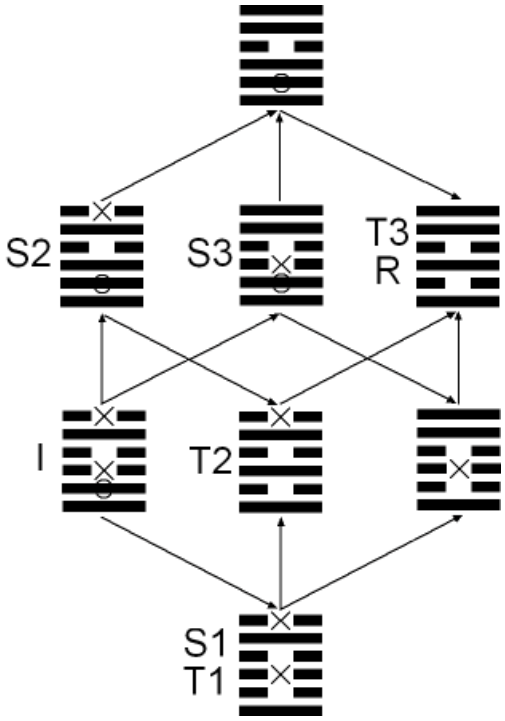

Figure 12: The Sublattice for 796876

I shall not consider the cases with more than three changing lines in this paper; the diagrams become increasing unwieldy. However, the reader can be assured that the formal analysis presented here is perfectly general and applies in all cases.

Conclusions

The Boolean lattice of hexagrams and its associated algebra provides a general structural theory for the symbols of the Yi Jing. In previous work I have shown how this theory can be used to analyse techniques from the traditional literature such as correctness and correspondence.[11] The work presented here extends the analysis to include a range of contemporary ideas. I have looked at some novel techniques from three contemporary authors for expanding the range of interpretive possibilities of the Yi Jing. Firstly, I have shown how these techniques relate to each other in their own terms; but more importantly, I have provided a single, general framework in which each of these ideas can be located as a special case.

Secter's Transition Hexagrams and Karcher's Steps of Change are of particular interest. Both are presented as techniques for breaking down a complex situation with multiple changing lines into a number of separate images that allow the components of the change to be analysed separately. These techniques seem to be at odds with each other because they generate different hexagrams for the same change. However, by using the idea of a sublattice derived from the Initial hexagram and its changing lines, I have provided a technique, which contains both of these ideas. Secter's technique identifies one particular path of change in its entirety, whilst Karcher identifies the starting point of every possible path. The sublattice of change shows all the possible paths.

The application of Boolean algebra to the Yi Jing has shown itself to be a very powerful analytical technique on a number of occasions. It is particularly interesting that contemporary authors, even those with no training in formal techniques, often invent methods that have natural algebraic characterizations. I believe this is because the natural structure of the Yi Jing, as a binary representation, conforms to the principles of Boolean algebra. Because of this, Boolean algebra is able to offer insights into the Yi Jing on many levels. That some of the traditional analyses of the Yi Jing, such as the diagrams in the *Zhouyi Tuishu Dadian* and Shao Yung's arrangements of the figures,[12] use principles from Boolean algebra in their construction is an indication that these ideas are embedded in the structural foundations of the Yi.

End Notes

[1] Andreas Schöter, 1998. "Boolean Algebra and the Yijing". This originally appeared in *The Oracle*, Volume 2, Number 7, Summer 1998, pp19-34, ISSN 1463-6220. It is also available as a download from http://www.yijing.co.uk.

[2] Edward Hacker, 1993. *The I Ching Handbook: A Pratical Guide to Logical and Personal Perspectives from the Ancient Chinese Book of Change*. Paradigm Publications, ISBN 0-912111-36-4. The Hexagram Flowers technique is described on pp94–96.

[3] For the sake of familiarity I shall use the English names for the gua from the Wilhelm/Baynes translation of the Yi Jing: *The I Ching or book of changes*, published by Routledge & Kegan Paul, 1983, ISBN 0 7100 9527 9. I shall combine the number from the King Wen sequence with the name for the complete hexagram reference.

[4] The current version of Secter's book is called *The I Ching Handbook: Decision-Making with and without Divination*, published by North Atlantic Books, 2002, ISBN 1-55643-415-4. Transitional Hexagrams are discussed on pp64–69 and Change Hexagrams are discussed from p163 onwards. This book was formerly published as *The I Ching Clarified: A Practical Guide*, by Tuttle in 1993. The earlier version did not contain the material on Change Hexagrams.

[5] To the best of my knowledge the "Steps of Change" technique first appears in Karcher's book *How to Use the I Ching*, published by Element Books in 1997, ISBN 1-86204-476-7. The Steps of Change method is described on pp24–26 of that volume. The material was then elaborated and given a sound formal basis in the joint paper "Tools For Change", co-authored by Stephen Karcher and Andreas Schöter. This

appeared in *The Oracle*, Volume 2, Number 12, January 2002. ISSN 1-463-6220. This joint paper is available as a download from http://www.yijing.co.uk.

[6] The presentation of lattice theory in this paper is kept to a very non-technical level and is done through the use of diagrams. The reader interested in some of the details of the mathematics behind the structures should refer to my earlier paper "Boolean Algebra and the Yijing" mentioned in Note 1 above, or to a standard text such as *Introduction to Lattices and Order* by B. A. Davey and H. A. Priestly, published by Cambridge University Press, 1990. ISBN 0 521 36766 2.

[7] From the Wilhelm/Baynes translation, p265.

[8] *Zhouyi Tuishi Dadian* "Encyclopaedia of Zhouyi Diagrams" published in two volumes in Beijing in 1994. Unfortunately, I do not have direct access to this material and I am indebted to S. J. Marshall for making scans of some of these diagrams available at http://www.biroco.com/yijing/scan.htm.

[9] Thomas Cleary, 1989. *I Ching Mandalas: A Program of Study for the Book of Changes*. Shambala Publications Inc. ISBN 0-87773-418-6.

[10] From a formal perspective, the sublattice describing a situation with a single changing line, shown in Figure 10, is structurally identical with the full lattice for a single line shown in Figure 4. Similarly, the structure of the sublattice describing the change when two lines are involved (see Figure 11) is identical to the structure of the bigram lattice shown in Figure 5. Further, when three changing lines are involved, the sublattice of change (see Figure 12) is structurally identical with the trigram lattice shown in Figure 6. If we were to extend the analysis to include four and five line figures the same would be true: a sublattice of change involving *n* changing lines is structurally identical with the full lattice for *n*-lined figures.

[11] Andreas Schöter, 1999. "Correctness and Correspondence". This originally appeared in *The Oracle*, Volume 2, Number 8, February 1999, pp25-37, ISSN 1463-6220. It is also available as a download from http://www.yijing.co.uk. Note that the paper "Tools for Change" mentioned in Note 5 above also includes an analysis of the nuclear trigram within the context of Boolean algebra.

[12] Shao Yung's diagrams are reproduced in many texts about the Yi Jing. An excellent account of his life and ideas, including reproductions of a number of his diagrams, can be found in *Transition to Neo-Confucianism: Shao Yung on Knowledge and Symbols of Reality*, by Anne D. Birdwhistell. Published by Stanford University Press, 1989. ISBN 0-8047-1550-5.