The Yijing as a Mathematical Metaphysics

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Introduction

In this paper I weave together a number of apparently distinct threads of investigation: the spiritual function of mathematics, the Image and Number (*Xiangshu* 象數) approach to the study of change, information theory and reality, divination and consciousness. The general background to this discussion is the idea that the study of mathematics forms a spiritual pursuit; that the investigation of abstraction through precise formal language provides, at the least, an analogue of transcendent reality. This was once a key underlying assumption of mathematics, but it is largely neglected in the modern world. The Image and Number approach to the study of Change embodies this ideal and asks us to take seriously the idea that, just as mathematics can describe the physical dimensions of the world, so too it can provide a language to investigate the psycho-spiritual dimensions.

In physics, vacuum polarization is explored. This started as a theoretical consequence of a mathematical description from quantum theory which, decades later, later received experimental verification. I suggest there is a strong parallel between this effect and the metaphysical progression from *Wuji* 無極 to *Taiji* 太極. This serves as the canonical example of the meeting point between mathematics, physics and metaphysics. I then look at comparisons between the three realms of Heaven (*Tian* 天), Earth (*Di* 地) and Humanity (*Ren* 人) as described in the *Dazhuan* «大傳» and the philosophical physics of David Bohm, in particular his key idea of the implicate and explicate orders and their relationship to each other and to consciousness.

The pivotal role of consciousness in the structure of the cosmos suggested by both the traditional and modern approaches then leads me to consider Chaitin's information theoretic perspective on randomness, especially the role of the cognitive faculties of the mathematician in determining patterns in data. The combination of Chaitin's ideas on pattern and randomness with Bohm's metaphysics suggests a way of understanding the Jungian notion of synchronicity as something more than a purely psychological phenomenon. Instead, this phenomenon can be seen as a direct effect of recognizing the integration of the human psyche into the broader fabric of the universal reality. In turn, this leads to a consideration of what the process of divination means in the context of the Western scientific mind set and, finally, how such a shift in perspective should lead us all to seek a harmonious way of being, both with each other and with the world which supports us.

Mathematics and Abstraction

Although contemporary mathematics is largely concerned with the complex technicalities of developing and extending the impressive edifice of proof and structure that has been built up over the generations, it is generally well accepted that the early history of the study involves a concern for matters we would call metaphysical, even spiritual. There was a mystery and, often, a mysticism surrounding numbers and the rules that governed them. This is true both in the Greek roots of Western mathematics and in the philosophical traditions of China. In the Greek tradition, this partly finds expression in the Pythagorian and Platonic conception of the reality of numbers, the

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idea that numbers have a real existence separate from, yet somehow connected with, the world of sensory experience. In the Chinese tradition, especially in connection with the study of Change, numerical mysticism is explored in the the School of Image and Number (*Xiangshu Xue* 象數學).

Shao Yong and Leibniz

One of the great exponents of the numerical study of Change is the Song 宋 dynasty philosopher and mathematician Shao Yong 邵雍 (1011-77CE) who developed a sophisticated theory relating numerical and symbolic representations to our capacity for knowledge of reality.[1] The underlying assumption behind the development of his ideas is summarized by Birdwhistell (1989 pp76-77):

> *Shao believed that every event and thing has a numerical aspect. Working from the numerical principles of those things that are immediately accessible to ordinary sense experience, one extends numerical principles to include all things and processes. Given the belief that the universe is regular and predictable, Shao assumed that a knowledge of the world's numerical structure would lead to further understanding of the universe and its process of change.*

This position is essentially the same as that taken by mathematical physics today.[2] Now, given that the realm of numbers is to serve as an explanatory mechanism for events in the phenomenal world, it is important that there is a clear connection between the abstract realm of mathematics and the world of concrete experience. This is not a trivial issue. It still raises questions for contemporary mathematicians, and Shao was well aware of the importance of providing a consistent description of this key relationship. Modifying the ideas Wang Bi 王弼 (226-249CE) connecting ideas, images and words, Shao said (translated Birdwhistell *ibid.*, p76):

> *If there are ideas, there must be words. If there are words, there must be images. If there are images, there must be numbers. After the numbers were established, then the images were produced. After the images were produced, then the words were clear. After the words were clear, then the ideas were manifest.*

Thus, he seems to be suggesting a direct connection from the entities in one realm to the entities in the other realm. Whether this connection is causal is not clarified, but it certainly carries a force of necessity. Numbers are the most abstract level of reality, able to represent the abstract structure implicit in our sensory reality. Everything else, images, ideas, words, events, follow from the abstract structure. The precise ontological status of this realm of number is a question that I will return to presently.

In the 17th century the mathematician Gottfried Leibniz (1646-1716CE) still professed a view of mathematics as a spiritual endeavour. For Leibniz "a universal truth underlay the diversity of human experience [and] that universal truth could be explained and proved mathematically" (de Fancourt, 1997 p156). The parallels between Leibniz's binary arithmetic, based only on 0 and 1, and Shao Yong's systematic arrangement of the *gua* 卦 are striking even now and must have come as a powerful confirmation of his views when Leibniz was first shown the arrangement by the Jesuit missionary Bouvet. For both Leibniz and Shao Yong, separated by centuries and thousands of

1 The parallels between Shao's thought and the ideas of Pythagoras are explored by Ding (2005).

2 Of course, we need to be wary of imposing our contemporary conceptions onto a mindset from an earlier age. However, restricting ourselves to Shao's theoretical writings, there do seem to be striking similarities.

miles, "the cosmos was created according to the binary system, which is reflected in all things and discoverable by human beings" (Ryan, 1996 p59). In particular, Leibniz came to believe that this system showed how God, which he saw entokened in 1, created everything from the void, entokened in 0.

The mystical and spiritual concerns expressed by Leibniz about the nature of the binary system are typically ignored or even derided in modern discussions of the history of mathematics. This reflects what we might call the secularization of mathematics in the modern age – a shift away from a concern with the metaphysical underpinnings of the subject as physical materialism has come to dominate the Western scientific mindset. However, there remain important connections between mathematical knowledge and spirituality. For example, Graham and Kantor (2009) describe how nineteenth century Russian mysticism became deeply intertwined with the development of the mathematical study of infinity. In fact, they suggest that it was the spiritual outlook of the mathematicians that allowed them to solve issues that had thwarted their more rationalist French colleagues. Most recently, perhaps, the philosopher and mathematician Merrell-Wolff said of mathematics that "our most abstract language is the best vehicle of ultimate truth" (cited McFarlane, 1995). I believe that this connection between mathematical abstraction and spiritual understanding is what makes the *Yijing* «易經» such a powerful tool across such a wide variety of both metaphysical and practical domains.

Mathematics and Reality

Echoing Shao Yong's concern to establish a basis for the connection between number and phenomenal reality is the recent focus on what is often described as "the unreasonable effectiveness of mathematics". That is, the fact that scientific theories, most especially in the domains of physics and engineering, make spectacular predictions and enable the construction of fabulous mechanisms entirely through mathematical reasoning. Wigner (1960 p1) says that "the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and […] there is no rational explanation for it." Wigner presents no solution to this apparent dilemma and, when revisiting the question two decades later, Hamming (1980) can shed no further light on the question beyond stating that our intellectual apparatus determines the results we find. I wish to suggest that the problem for both of these writers, in relation to this, is that they did not think radically enough about the issue. Hamming explicitly states that mathematics is the product of the human mind, and I believe it is this perspective that, in part, makes the most obvious answer so elusive.

There are a variety of possible views around the "reality" of mathematics. Understanding mathematics as a purely mental construct is the *idealist* perspective. An idealist may be completely subjective, where each individual has their own mental construction of mathematics, or inter-subjective, where our common cognitive substrate gives us a shared mathematical construct; but they all agree that without minds there would be no mathematics. The counter position to this is *ontological realism*, where the subject matter of mathematics, numbers, sets, and so on, are taken to exist independently of the mind of the mathematician.[3] The exact nature of this objective existence can vary from, at one extreme, the position of hypothesizing an actual realm of existence where these mathematical objects reside to, at the other, simply asserting the objective nature of mathematical truth (see Shapiro 2000 for an extensive discussion of these matters).

3 Following its roots in Greek mathematics, this view is sometimes referred to as Platonism.

However, ontological realism is not enough, on its own, to solve the problem of the "unreasonable effectiveness" of mathematics. In fact, on its own, it merely raises the question in a different form. Consider the view of the prominent contemporary mathematician Roger Penrose, expressed in his encyclopedic review of physics (Penrose, 2005 pp17-21). For Penrose, there are three distinct realms that together form reality, which he calls the world of Platonic mathematics, the physical world, and the mental world. These worlds are not separate, but have "mysterious connections": the physical world is somehow governed by mathematical principles, the mental world is dependent on certain physical phenomena, and the mathematical world is somehow perceived by some forms of mental activity. The precise nature of the connections between the various worlds has simply replaced the problem of how mathematics is effective at describing the world.

Max Tegmark (2007) takes a different position and, as a result, completely sidesteps these problems. For Tegmark, as an ontological realist, mathematical structures are not the product of the human mind, instead we discover them; our only invention is the notation we use to describe them. However, he pushes this view to its logical conclusion with what he calls the *mathematical universe hypothesis*: "our universe is not just described by mathematics – it is mathematics … our physical reality is a mathematical structure". Of course, if the universe is a mathematical structure, then it is not at all mysterious that mathematics should be so useful in describing it. If the universe is a mathematical structure, then everything within it, our consciousness included, is also a mathematical structure.[4] This view also finds expression in the idea that the constantly changing universe is essentially the result of an ongoing quantum digital computation (see, for example, Lloyd 2006 and Seife 2006). This view is not meant as a metaphor by its proponents, but as a literal, objective truth about the fundamental nature of reality. From this perspective everything is, at root, information, and events are the results of the computational processes that occur when the information from two or more 'separate' components interact. Shao's solution to the problem of connecting numbers, images, ideas and words finds a strong echo in the contemporary information theoretic approach to meaning and reality. Lloyd says (2006 p27):

> *Although meaning is hard to define, it is one of the most powerful features of information. The basic idea of information is that one physical system – a digit, a letter, a word, a sentence – can be put into correspondence with another physical system.*

All physical properties have units of measurement. For example, science measures length in meters and mass in grams. So too, information has a unit of measure: the *bit*. This is the smallest amount of information that anything can carry and it is simply the distinction between 0 and 1. Therefore, because everything is, at root, information everything can be accurately expressed using the binary notation of 0 and 1.

These radical views on the nature of reality have important consequences below, when we consider the interaction between the unfolding of physical phenomena, our consciousness, and the symbolic language, the *gua* 卦, of the *Yijing*.

4 Although I share Tegmark's basic position regarding the nature of reality, my view of the nature of mathematical structure is somewhat different. Rather than being fixed, eternal and unchanging, as a Platonist would hold, I believe the mathematical structure itself evolves over time as aspects previously implicit become explicit. This happens as a the result of the computation which occurs when information is processed through quantum interactions.

The Yijing as a Notation System

Even if mathematical structure is the root of our reality, and has some kind of objective existence separate from our conceptions, the notation that we use to express that reality is an invention of the human mind. This makes notation, as suggested by Long (1999 p21), our "primary interface between minds and realities" and, as such, its development and refinement deserves a great deal of attention. David Bohm (1980) suggests that the most pervasive problem facing the modern world is fragmentation. This fragmentation exists at all levels of our world view and culture because our thought itself is fragmented. Although division of the world into categories and entities is a practical necessity, it is important to realize the nature of the relationship between thought and the world. Fragmented thought leads to a systemic confusion where the divisions in thought are mistaken for real divisions in the world. This makes it difficult to recognize the true nature of problems, and it is therefore impossible to find appropriate solutions. Because of this, healing the fragmentation of thought is a crucial first step if we are to solve any of the many serious problems we are currently facing. One of Bohm's suggestions for helping to first see, and then heal, this fragmentation, is close attention to the use of language and thought. He introduces what he calls the "rheomode" for language, where he seeks to develop a mode of speaking/thinking which recognizes the nature of reality as a seamless, connected flow and actually draws attention to the function of language in its actual use.[5] He suggests that each of us should become an active experimenter with our use of language and thought.

Long (2005 p2) describes notation systems as "a cognitive technology that in fact has been essential for the development of modern civilization and the modern mind" and that such systems (*ibid.* p4) "enable its users to see and utilize facets of reality that they literally had not been able to see before." Thus, notation systems are extremely powerful tools. I further wish to suggest that, considered as a notation system, the *Yijing* can play an important role in Bohm's project of developing language modes and conceptual tools which recognize the nature of reality as a continuously interacting flux.

The Symbols of Change

Long (1999 p22) provides a description of the different levels of a notation system. This is summarized in Table 1 below. The numbering of the levels and the additional column relating the levels directly to the *Yijing* are my own.

Level	**Description**		**Yijing**
4	Fashions of Use	Style	Schools
3	Testing Rules	Acceptance	Divination
2	Combination Rules	Syntax	Lattices
1	Symbol Set	Tokens	Gua
0	Abstraction Space	Semantics	Abstract Jin

Table 1: Levels of a Notation System

5 As a brief aside, I suggest that a comprehensive development of the rheomode could potentially provide an excellent basis for interpreting the texts of the *Yijing*.

At level 1, the tokens, the *gua* 卦 of the *Yijing*, are the most visible aspect of the Book of Change when considered as a notation system. The symbol set includes single lines, bigrams, trigrams and hexagrams. However, in the general case it should also be taken to include four and five lined *gua* and symbols of arbitrary size. There are a number of aspects of the *gua*, considered purely as tokens in a symbol system that I wish to mention. Firstly, these symbols have an important property: they are binary. That is, the *gua* are already expressing their information in the most fundamental units of information theory – the bit. This property makes them extremely flexible as a form of notation. It has been reported (Combs & Holland, 2001 pXIX) that the psychologist Jung and physicist Pauli speculated that the counting numbers form the lowest level of archetypal structure.[6] It is of great interest, in the context of this discussion, that they suggest that a mathematical structure could form the basis of the psychoid archetypes; however, I wish to suggest an alternative mathematical structure for this basis. We will see below that the notion of polarity sits at the foundation of the metaphysics of the *Yijing* and this, I contend, is the obvious choice for the base archetype. Thus, as patterns of polarity, the *gua* are not only fundamental from an information theoretic perspective, but also from the perspective of psychoid archetypes.

Next, although the precise details are today called into question, the suggestion by Miller (1956) that human short term memory can process, on average, between 5 and 9 items (the so-called "seven plus or minus two" hypothesis) remains a good rough guide to our immediate cognitive capacity. With six lines, the hexagrams fall easily into this range, making them a digestible packet of information for the human mind to process. Of course, the question then arises as to why six lines, and not seven or eight? I believe that part of the answer to this question lies in the third important property of the symbols, and that is the high factorability of the number six. That is, 6 can be divided by both 3 and 2, which are significant numbers in the metaphysics of the *Yijing*. This is the only number in the range of short term memory that has such a property. The next such number is twelve, which is too large to be easily processed.

It is the combination of these three properties that makes the symbols of the *Yijing* so compelling: i) They are binary, already expressing the fundamental units of information; ii) they are easily processed by our cognitive apparatus; and iii) they have the right factors for their internal divisions to fit directly into the underlying metaphysics.

The Abstraction Space for Change

Level 0 describes the referents of the tokens. In Long's analysis the tokens do not refer directly to the real world, but instead to an *abstraction space*. This abstraction space then refers to the real world or even, in some cases perhaps, generates the real world. This indirection gives notation systems a great deal of flexibility: different abstraction spaces allow us to see the real world from very different perspectives. Long says (1999 p26) that "the abstraction spaces which are reified in a notational system refer to something that is independently discoverable by different minds." And that abstraction spaces are (*ibid*. p27) "the foundation of notational systems: the existence of a useful notational system implies something about the nature of reality."

So, given its key role in a notation system and the implications carried for the nature of reality, it is vital to identify the abstraction space of the *Yijing* if we are to make sense of it as a notation system.

6 In Jungian psychology an archetype is an essentially irrepresentable basic form that bridges the physical and mental domains and provides common organizing structure for the content of thought. Because archetypes are not purely psychological, Jung called them *psychoid*. See, for example, Jung 1960.

Perhaps this space has already been identified for us – Wang Bi (translated by Lynn, 1994 p31) gives the following description:

> *The words are generated by the images, thus one can ponder the words and so observe what the images are. The images are generated by ideas, thus one can ponder the images and so observe what the ideas are.*

Notice how Lynn translates Wang's description of this relationship: the ideas *generate* the images, which *generate* the words. In the analysis of the *Yijing* as a notation system the images are the tokens of the system, the *gua*. Following this, the abstraction space would correspond to the ideas. However, this does not help us much. What, exactly, is meant by "idea" in this context? Instead, I wish to explore how patterns of energy, as directly represented by patterns of *Yin* 陰 and *Yang* 陽 in the *gua* could form a viable abstraction space.

The term *jin* 勁 is used in the martial arts to refer to the various ways of expressing energy through the body. These cover a wide range of possibilities, for example Yang (1996 pp106-107) gives 53 distinct *jins* for *Taiji Quan* 太極拳. Thus, *fa jin* 發勁 is the explosive release of energy into an opponent; *na jin* 拿勁 is locking energy which involves locking joints to control your opponent's movement; *hua jin* 化勁, neutralizing energy, is using softness to overcome the opponent's attack by redirecting their energy; and *ting jin* 聽勁 is the sensitive application of one's own energy to listen to an opponent's intentions. Now, given that the trigrams are often traditionally used to represent eight of the key *jin* in *Taiji Quan* (see, for example, Huang, 1984 p90 or Schöter, 2004a), I wish to suggest that a suitably generalized notion of *jin* – what we might call *abstract jin*, *chouxiang jin* 抽象勁 – form a good candidate for members of the abstraction space of the *Yijing*. Such abstract *jin* would need to include psychological and spiritual as well as physical energies, identifying the key properties of energetic expression that can be understood and applied across a wide variety of domains.[7]

Long (1999 p23) allows for the possibility that the same tokens might appear in distinct notation systems. Therefore, I also wish to suggest that a key feature of the *Yijing* is that it actually embodies a family of potentially distinct notation systems whose detailed interpretation depends on the precise abstraction space that is chosen for the *gua*. However, I also suggest that each of those abstraction spaces would be specializations of the most general space of abstract *jin*. An interesting example of the application of the energetic images associated with the *gua* that, I believe, fits with the idea of abstract *jin* can be found in Suler (1993 Chapter 9). He uses the traditional attributes of the trigrams, the *Yin*/*Yang* patterns in the lines in individual *gua*, and the relationships between *gua* to describe aspects of the dynamics of the psychotherapeutic process.

The Other Levels of a Notation System

The syntax of a notation system, how symbols are composed and combined, forms level 2. In the *Yijing* this includes, for example, elements of internal structural composition such as the interlocking trigrams (*hu gua* 互卦) and containing trigram (*bao gua* 包卦), as well as the various forms of opposition such as the overturned hexagram (*fan gua* 反卦) and laterally linked hexagram

7 Given recent work that seems to show that abstract thought is, in fact, strongly correlated with physical movement in our bodies (Anathaswamy, 2010) this might actually be a very useful starting point. Bohm (1992 p42) takes this a step further, saying that "thought is not merely the intellectual activity; rather it is one connected process which includes feeling and the body, and so on. Also, it passes between people – it's all one process all over the world."

(*pang tong gua* 旁通卦). Nielsen (2003) is a comprehensive source of information about these traditional syntactic aspects of the *gua* in English. It is also possible to apply a wide range of contemporary algebraic techniques to the *gua*. This can be done to provide a precise formal description of some of the traditional syntactic manipulations (for example, Schöter 1998 and 1999). It is also possible to apply these techniques to generate and analyse novel structural aspects of the *gua*. For example, in Schöter 2004c and 2005 I apply Boolean lattice theory to provide a detailed structural description of the unfolding of Change. Much of my previous research has focussed on this level of notation systems, and I refer the interested reader there for more details on this aspect.

Long describes level 3 as defining the rules we use to validate the expressions that are generated at level 2 of a notation system. Generally we could consider this as, firstly, the application of the notation system to its intended problem domain in its intended manner and, secondly, the evaluation of that application, including the criteria of such evaluation. He says (1999 p23) that "each broad kind of disciple has its own distinct rules for acceptance." For the *Yijing* the acceptance of the notation system depends, on the one hand, on its successful use as a language for clarifying philosophical and metaphysical discourse and, on the other, on its practical application as a tool for individual spiritual development and for providing insight into resolving personal problems. This covers a great deal of territory but nonetheless accurately reflects the historical range of application of the book.

The final level of a notational system is the stylistic level, involving what Long describes as "the fashions of use". I have little to say about this level in this paper, except to note that the different schools of interpretation such as *Meaning and Principle* (*Yilixue* 義理學) and *Image and Number* (*Xiangshuxue* 象數學) arise at this level.

Physics and Metaphysics

I am fond of quoting Russell (1917 p20), where he characterizes metaphysics as "the attempt to conceive the world as a whole through thought." To this we must add Bohm's proposition (1980 p71) that such comprehension "be considered an art form, like poetry, which may dispose us toward order and harmony in the overall 'dance of the mind' (and thus in the general functioning of the brain and nervous system)."

Above I suggested that the abstraction space for the *Yijing* could be considered as being composed of abstract *jin*. However, these patterns of energy do not exist in isolation; they exist in the context of a background field described by the underlying metaphysics of Change. In this section I shall explore some of the connections between these metaphysical expressions and ideas from physics. As physics is the canonical example of the successful application of the language of mathematics to describe the world, it is very interesting that we can draw some strong parallels between some of its concepts and the metaphysics of the *Yijing*.

Vacuum Polarization

The paired appearances (*liangyi* 兩儀) of *Yin* 陰 and *Yang* 陽 provide the foundation of the *Yijing's* notation system – both for the recursive, graphical development of the actual symbols and, more generally, for the abstraction space itself. The key relationship between *Wuji* 無極, *Taiji* 太極 and

Yin and *Yang* is spelt out in the opening lines of the "Explanation of the *Taiji* Diagram" (*Taijitu Shuo* «太極圖說») by Zhou Dunyi 周敦頤 (1017-73CE). This says (translated Adler, 2009 p2):

> *Non-polar and yet Supreme Polarity!*
> *The Supreme Polarity in activity generates* yang*; yet at the limit of activity it is still. In stillness it generates* yin*; yet at the limit of stillness it is also active.*
> *Activity and stillness alternate; each is the basis of the other.*

We can visualize this development graphically through the simplified diagram given as Figure 1. Here we see a bi-directional connection between *Wuji* and *Taiji*, indicating their interconnected relationship: *Wuji* is undifferentiated, but nonetheless contains the potential for differentiation to arise. Once differentiation occurs, then the two poles, *Yin* and *Yang*, must be identified and characterized. This progression lays the foundation for the recursive generation of the *gua* by showing how *Yin* and *Yang* arise, via *Taiji*, from *Wuji*. *Wuji* is the primordial, limitless void, it is the unpolarized state before any phenomena arise. In contrast, *Taiji* is the "supreme polarization", the arising of something from nothing.[8]

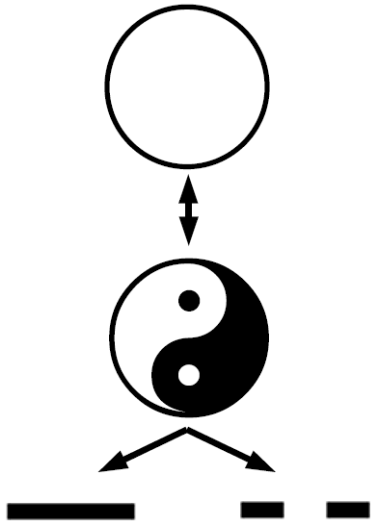

Figure 1: Wuji, Taiji, Yin and Yang

This description is very interesting from the perspective of quantum mechanics, particularly with respect to the phenomena of virtual particle pairs and vacuum polarization.[9] Briefly, energy and lifetime of a particle, or group of particles, form a complementary pair of properties: the more precisely one knows the energy of the particles, the uncertain one is of the lifetime, and *vice versa*. Therefore, applying Heisenberg's Uncertainty Principle to these properties means that it must be the case that the apparent vacuum is actually constantly giving rise to short-lived pairs of particles and anti-particles; for example, an electron-positron pair can arise spontaneously out of the quantum vacuum. The energy required to create the pair of particles is repaid by the energy released in their mutual annihilation. The combined uncertainty of the energy and lifetime of these pairs is so small that the Uncertainty Principle renders them unobservable and this in turn ensures that the conservation of energy is not violated. As Barrow (2001 p233) says "the quantum vacuum can be viewed as a sea composed of all the elementary particles and their antiparticles constantly appearing and disappearing." I suggest that this is a modern rendering of the idea that *Taiji* arises from *Wuji*: *Wuji* describes the quantum vacuum – an apparently empty, but infinitely creative state; then *Taiji* can be seen as the closely bound pair of a particle and its antiparticle that arise spontaneously from the vacuum.

Although *Yin* and *Yang* exist in *Taiji* they are still tightly bound together, and the final stage in the metaphysical development shown in Figure 1 can be seen as the arising of *Yin* and *Yang* as individual, yet connected pair, of principles which interact to create observable phenomena. Cheng

8 As Adler (2009 p24) notes, whilst the usual translation of *Taiji* as "Supreme Ultimate" catches its colloquial meaning, it "completely misses the significance of the term" in relation to *Wuji* and *Yin*/*Yang* as polarity being the most fundamental principle of ordering reality. Earlier (p11) in he describes *Taiji* as "the principle or pattern of polarity", equating *Taiji* 太極 with *li* 理. It is because of the verbal, rather than the nominal, mode of this expression that I prefer the translation "supreme polarization".

9 The full details of this phenomenon are complex and would take us too far afield here. The interested reader can find a digestible account in Barrow (2001 pp230-235).

Chung-ying (personal communication, email 25 May 2011), notes that the quantum description of this phenomenon is "a continuous process of 'immediate birth and immediate death'" and raises the question of "how this process could eventually bring out a process of differentiation of things based on the initial action of *yin* and *yang* which cooperate with, rather than eliminate, each other." To answer this question, the time scale of vacuum polarization needs to be considered. Firstly, it is not strictly accurate to describe this as "immediate" birth and "immediate" death: the mathematics provides a precise bound on the time interval for which the particle/anti-particle pair can exist, which is inversely proportional to the amount of energy required to bring the pair into existence. Now, if the virtual particle pair is charged then, for the duration of their existence, they create an electromagnetic field between them. I would therefore suggest that, for the duration of its existence, the virtual particle pair is manifesting a *Yin*/*Yang* interaction.

Cheng also asks if there is "a principle of non-parity which makes the event of generating or *sheng* 生 possible?" and if so, how such non-parity is possible? In asking this question in this context Cheng touches on a question that remains troubling to physicists. One tentative explanation for the "big bang" view of the creation of the universe is that it was an instance of vacuum polarization. Then, everything that is, is simply an aspect of a quantum fluctuation that will, eventually, collapse back into the void. But, according to the mathematical theories, there should be equal amounts of matter and antimatter in the universe. Cheng's question then, amounts to asking why there is more matter than antimatter, and to that question no one currently has an answer.

Bohm's Holographic Universe

Bohm (1980 p181) says "not only is undivided wholeness implied in the *content* of physics (notably relativity and quantum theory) but also in the *manner of working* in physics." As a result of this he argues for a radically new order aimed at exploring the connection between theoretical frameworks, observations and instrumentation which, he says, cannot be coherently regarded as separate from each other. He proceeds by analogy with the hologram which captures a three-dimensional image of a solid object on a two-dimensional photographic plane by means of recording the interference patterns in coherent light. Each portion of the hologram contains information about the whole image. Thus, when we look at the whole image, we are able to see the original object from various angles as we change our view point on the hologram. However, if we take only a portion of the hologram, we see not a part of the original object, but rather the whole original object but with reduced brightness and from restricted viewpoints. So, each the pattern in each region of the image encodes aspects of the whole object and, conversely, each part of the object is encoded across the whole pattern of the hologram. This is clearly very different from an ordinary photographic image, where each portion of the image merely encodes a portion of the object and taking only a portion of the image shows us only a portion of the original object.

Bohm conceives of the whole of reality in such a holographic manner. That is, each part of reality contains information about the structure of the rest of reality "enfolded" within it. This enfolded structure is what Bohm calls the *implicate* order. The notion of implicate order is to be contrasted with *explicate* order: the aspect of reality that is immediately available to our senses and instrumentation. From one perspective, parts of the implicate order are carried as information. For example, the patterns of vibration in light waves enfold information about the objects they have interacted with on their journey through the universe. More generally, the information structures of the implicate order are carried by the *holomovement* – a generalization of all the possible carriers of implicate order. The explicate order, our manifest reality is continuously unfolding from the

holomovement of the implicate order and then re-enfolding into it. In this picture, the implicate order is primary. What we see as causal links between events in the explicate order are, more properly, to be seen as connected unfoldings from the implicate order, related by deeper principles which are to be expressed in terms appropriate to the implicate order, rather than the gross mechanical rules of classical physics.[10]

Consciousness plays a key role in this picture of the structure of reality. Bohm uses the example of our perception of music. The notes of a melody arrive sequentially, separated in time, but it is the reverberation of earlier notes continuing in our consciousness that provides the context for the understanding of the current note. The earlier elements of the music, although they have ceased to exist in the explicate order, have been enfolded into the implicate order of our mind, whilst the perception of the immediate sensation of the new note forms an explicate order of thought. It is the interaction between these two aspects that actively creates the structure of the piece. Thus, our bodies, as stable, extended physical structures are clear members of the explicate order, whilst our mind, able to enfold and unfold information, is in the implicate order.

I have previously suggested (Schöter 2004b) that we can make a strong parallel between the picture of reality given by Bohm and the structure given in the *Yijing* based on the traditional realms of *Tian* 天 (heaven), *Di* 地 (earth) and *Ren* 人 (man). Consider the following lines from the *Dazhuan* «大傳» translated by Wu (1991 p263):

> *In the heavens images are completed.*
> *On the earth bodies are formed.*
> *...*
> *Qian knows the great beginnings.*
> *Kun makes and finishes things.*

The trigram associated with heaven is *Qian* 乾, the Creative; this is the source of all movement, and generates the patterns which events follow. In contrast, the trigram associated with earth is *Kun* 坤, the Receptive; this provides a material substrate in which the unfolding of the Creative patterns can actually take form. The parallels between the implicate order as *Tian* and the explicate order as *Di* are clear. Further, in the traditional metaphysics man, *Ren*, arises between, and serves to connect, heaven and earth, which is exactly how consciousness functions in Bohm's picture.

This relationship is represented directly in the structure of the hexagram as shown in Figure 2. I mentioned the factorability of the six lines of the hexagram above, and here we can see this graphically. Consider the major division into two domains first: the top three lines of the *gua* form the upper trigram, which represents the pattern of energy in heaven, the implicate order; the bottom three lines form the lower trigram, which represents the pattern of energy in earth, the explicate order.[11]

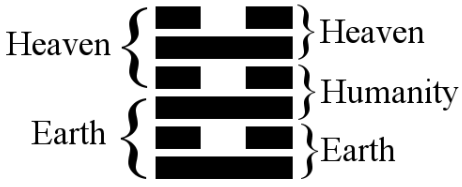

Figure 2: The Domains of Change

10 Bohm actually suggests a whole hierarchy of implicate and super-implicate orders, each unfolding into the order below until the phenomena become manifest in the explicate order. The simplification, in this paper, of talking only of *the* implicate order does not detract from the essence of his view.

11 In the hexagram, these two domains are distinct and do not overlap. However, the traditional relationship of correspondence, *Ying* 應, describes a dynamic connection between the patterns in the two domains.

Now consider the ternary division shown on the right of the diagram. Here the middle two lines are assigned to the domain of man. Thus, the bottom line of the top trigram (heaven) and the top line of the bottom trigram (earth) combine together to give form to humanity, where the function of the mind acts in the implicate order and the body is a manifestation in the explicate order.

I have now presented two key metaphysical aspects of the foundation of the *Yijing*. The first described how the basic polarization of experience arises spontaneously from the undifferentiated void. The second described the division of reality into distinct but interacting domains. These two ideas contribute to the conceptual foundation of the abstraction space of the *Yijing* – the space where the *gua* find their referents. In both cases we found ideas in modern physics that provide striking parallels. I suggest that this lends weight to the assertion that the underlying symbolism of the *Yijing* is, in a strong sense, universal, and can therefore provide a useful notation system for exploring a wide range of issues.

Information Theory and Divination

For Zhu Xi 朱熹 (1130-1200CE) the act of divination is at the heart of understanding the *Yijing*; describing the original meaning of the work as "not in its various layers of text, but in the oracular function of the hexagrams." (Adler, 1990 p1). The ancient understanding of the manner of divination's function relied on the agency of *gui-shen* 鬼神 (ghosts and spirits). However, although he used this terminology, Zhu interpreted these as essentially impersonal: one the one hand, they were "nothing more than the waxing and waning of yin and yang" and on the other, were also "traces of the creative process" (Adler, *ibid.* p22-23). Thus, divination is not concerned with the supernatural, but with the more subtle aspects of the natural. In Zhu's view divination is a technique for self-cultivate and requires a quiet mind that can listen to the waxing and waning of *yin* and *yang*; only if one is peaceful and concentrated is it possible to hear the response of the *gui-shen*.

Divination is a difficult practice for the modern scientific framework to accommodate. Perhaps the best we might hope for from the perspective of classical science is an explanation based in the psychological aspects of interpretation and significance. I have previously explored divination in terms of the dynamics of a conversation between the symbolic content of the *Yijing* and the diviner (Schöter, 2004b). Here I wish to revisit some of those ideas, delving a little deeper into the information theoretic perspective and, eventually, connecting these issues with the metaphysical foundations already described. The aim is to provide scope for a stronger explanation of divination that finds a basis in the interaction of the implicate and explicate orders.

To begin with I shall define the process of divination as the interpretation, in relation to a given situation, of the symbolic output of an unpredictable process.[12] Along with the creative act of interpretation, chance and predictability are therefore at the heart of divination.

Pattern and Randomness

It is significant that modern mathematics has very little to say about the causal source, or lack thereof, of randomness. Instead, it concentrates on determining whether the symbolic output of any given procedure could be described as random, regardless of the nature of the generating procedure. Chaitin's information theoretic approach to randomness is of interest here (Chaitin, 1975).

12 Note that, on this view, divination is not about predicting the future; it is about revealing something hidden in the present.

Imagine I was in the habit of throwing a coin 18 times in succession and recording the result. On one occasion I got:

HTHTHTHTHTHTHTHTHT

And then I did it again and got:

TTHTHHTTTTHTHHHTTH

Psychologically, it is hard to accept that the first sequence is the result of a random process. This is because we can *see* the pattern clearly, and the chance of that pattern emerging by chance seems so unlikely. But there are 262,144 (2^{18}) possible sequences that can be generated by tossing a coin 18 times and each of them is equally likely; the first pattern is no more or less unlikely to occur than the second.

The information theoretic analysis of randomness is concerned with detecting patterns in the data. The first sequence shows an obvious pattern which can be expressed succinctly as HT×9. Mathematically then, it must be deemed a non-random sequence. However, the second sequence does not show an obvious pattern and, mathematically, it would therefore be taken to be random.[13] And yet both sequences have been generated by the same source. But, as Chaitin says, the "provenance of the series" is not enough to certify that it is random. He says (*ibid.*):

> *If origin in a probabilistic event were made the sole criterion of randomness, then both series would have to be considered random, and indeed so would all others, since the same mechanism can generate all the possible series. The conclusion is singularly unhelpful in distinguishing the random from the orderly.*

Thus, *distinguishing* whether a particular sequence is random or not is at the crux of the information theoretic approach, and this must be largely determined by the ingenuity of the mathematician doing the looking. If they can find an applicable description of the pattern, then they must declare the sequence as non-random. Only if they cannot find a description can they *provisionally* declare the sequence as random. Note that such a declaration can only be provisional because it is always possible that a more insightful analysis might uncover an applicable description of the pattern.[14] So, from a mathematical perspective, non-randomness is essentially a relationship between the source data and some well-defined and concise mathematical structure, and not a result of the causal origin of a sequence. The ability to recognize the relationship is then a perceptual act on the part of the mathematician.

Perhaps this seems backwards: surely both sequences are actually random, because we know they were generated by coin tossing; the first one merely *seems* non-random because we can see an apparent pattern in the data. Information theory says it is, in fact, the other way around. The first sequence is actually non-random *because* there is a concise pattern that describes it. We could say

13 On this analysis, randomness is also related to compressibility. A non-random sequence can be compressed without loss; that is, described in a succinct way. A random sequence cannot be significantly compressed without loss. Data compression is an important study in computing, which explains why information theory takes this approach to randomness.

14 Chaitin notes that it is, in principle, impossible to tell whether the sequence actually is random or only appears to be random. This result is connected to Gödel's famous incompleteness theorem and thus the nature of randomness seems to be a fundamental, information theoretical foundation of mathematics.

that information theory contrasts randomness with pattern, whereas typically we would tend to contrast randomness with predictability. We mistake the nature of randomness as residing in the process of tossing of the coins whereas the randomness really resides in the results of that process. What actually resides in the tossing of the coins is the *unpredictability* of the outcome of each event. Whether the effectively unpredictable results are random or not is a question that can only be determined after the process is complete.

Reflect on this characterization of randomness in the context of divination. Mathematically, the non-randomness of the result lies in the perception of an underlying pattern. Now, in divination the significance of the result lies in the perception of a connection to the originating situation. In both cases the source data is taken as a given, and the focus is on the identification of relevant structure. With this striking parallel in mind, it is interesting to consider that the practice of divination frequently makes use of explicitly unpredictable sources for the generation of the data.

Synchronicity and The Implicate Order

Combs and Holland (2001 p152) describe systems such as the *Yijing* as being a form of "active divination". The contrast is with the interpretation of omens and similar events, which happen regardless of the actions of the individual, but which are interpreted relative to the individual's situation. With the *Yijing*, however, the individual actively asks a question and then uses a deliberate technique to generate a symbol which is taken to encode an answer to the question. Even in the context of the preceding metaphysics, we are left with the question of the nature of the connection between, on the one hand, the situation that gives rise to the act of divination and with respect to which the symbols are interpreted and, on the other hand, the actual outcome of the divination process. More bluntly, how can the resulting symbols, generated by the unpredictable technique, be relevant to the problem? Answers to this question can take a variety of forms.

As already mentioned, the weakest answer, in terms of connection, is to simply say that the relevance lies entirely in the act of interpretation. This is similar to the information theoretic approach to randomness discussed above. In that case, a sequence is considered non-random if an analyst can find a compact description of the sequence, otherwise it is random. In terms of divination, the symbolic answer is relevant to the extent that a suitable interpretation can be given in relation to the original situation. The connection, in this picture, lies entirely in the mind of the diviner. Whilst such a picture raises interesting psychological questions, it is ultimately unsatisfactory from the perspective of the metaphysical framework.

Jung's principle of *synchronicity* is well-known in this context (Jung, 1955). Starting from the premise, already acknowledged in quantum physics, that causality is a problematical phenomenon, Jung suggests that there must be other principles that can serve to connect events in certain circumstances. To that end, he proposes synchronicity as an *acausal* connecting principle. The usual definition of this phenomenon is as "meaningful coincidence" – two or more events happen to occur, sometimes more or less contemporaneously, sometimes separated in time, and a significant connection is seen between them.[15] Of course, it could be argued that this explanation is still predominantly psychological: the significance is in the mind and perhaps the coincident events have no other connection. However, we have already seen in the metaphysics of both the *Yijing* and Bohm's physics that the explicate physical order, consciousness, and the implicate order of pattern

15 As a brief linguistic aside, it is interesting to consider the Chinese term for coincidence: this is *qiao he* 巧合 which, taking the literal meanings of the individual characters, could also be interpreted as an "ingenious combination".

are woven from the flow of a single fabric. Further, in Bohm's view, what connects events and things should ultimately be expressed in terms of the implicate order, not the explicate: things which have the same degree of enfoldment are *synordinant*, and so will unfold and become explicate together (Bohm, 1980 p194). But this notion of "together" is neither necessarily synchronous in time nor necessarily local in space – the implicate order can, in principle, unfold synordinate events at distinct spaciotemporal locations.

Because the mind is, in part, participating in the action of the implicate order it is conceivable that one could perceive synordinace even when spaciotemporal locality is lacking. This is, or course, sensitivity to a subtle, natural phenomenon, and therefore could be seen as listening to the *gui-shen*. Adler (1990 p17) notes that divination is to be explained in terms of *shen* (spirit) which enables both the mind and the unpredictable technique of the *Yi*[16] to respond to the underlying patterns of transformation in *qi* 氣 and that "as a characteristic of [*qi*] itself *shen* refers to the coherence of certain phenomena in which coherence is not empirically observable by the ordinary mind." In the context of the current discussion coherence which is not empirically observable could easy be interpreted in terms of synordinant events in the implicate order. Thus, Bohm's physics offers a potential explanation for the non-local, acausal phenomenon of synchronicity that weaves the psychological and the physical together into a single framework.

Morphic Resonance and Divination

Nonetheless, we are left with an explanatory gap. Even if we take on board implicate synordinance as the basic structural property of reality and allow that consciousness has some direct access to this ordering, we are left wondering how a series of unpredictable events can result in symbols that are relevant to the situation being divined about. That is, why, for any given act of divination, should we expect the patterns resulting from tossing some coins to be synordinant with the particular situation under consideration? But this is exactly what Jung suggested. Combs and Holland (2001 p153) say that Jung first introduced the concept of synchronicity in his memorial address for Richard Wilhelm in 1930, suggesting that it is the very unpredictability of the mechanism that allows the resulting pattern of the coins to reflect the broader patterns in the situation under consideration. But there is more to it than this: as Combs and Holland are aware, the attitude of the person making the divination is also important. Confirming most people's experience, they observe that bringing an attitude of respect and reverence to the act of consulting with Change is crucial. This echoes Zhu's admonishment to be "extremely dignified and extremely reverent" when consulting the *gui-shen* through divination (Adler, 1990 p23). This attitude is a deeply significant part of the process, helping the mind to become sensitive and responsive to Change.

The metaphysical framework outlined above allows for the mind to interact directly with the implicate order. By bringing the mind into a suitable state, its deep patterns may be able to imprint directly into the implicate order and therefore effect the resulting events that unfold into the explicate order. This effect is clearly not direct – it does not seem possible to directly control whether a coin lands heads or tails through thought – but a divination relies on the overall pattern of the coins, not the individual tosses, and the resulting overall pattern is indirectly interpreted as a pattern of change.

16 The technique most usually taken to have spiritual content is the milfoil stalk. However, I am suggesting that it is the random nature of the process, not the physical material used, which carries the spiritual charge.

The idea of the *morphic field* is relevant here (Sheldrake, 2009). In modern physics everything can be interpreted in terms of fields: electromagnetic fields are used daily in our radio and television transmissions, the receiver in the set resonates with the vibrations of the transmission to amplify its signal. But even particles such as electrons are to be understood as localized excitations of an underlying field, and the holomovement, discussed above as the carrier for the implicate order, is a vast collection of interacting fields. Sheldrake extends this idea to every aspect of reality. Thus, the mind itself generates a field that carries information about its intent and content. In this model then, the field of a mind, which already interpenetrates with the implicate order, could enter into resonance with those fields, resulting in a mutual influence where the deep concerns of the mind affect the content of the implicate order in such a way as to allow the unpredictable technique to express relevant pattern as it unfolds into the explicate order through the act of the divination.

Of course these ideas are highly speculative. Not only does Bohm's view of reality lack empirical support, but it is even difficult to see what experimental techniques could be applied to falsify it. Further, Sheldrake's work is extremely controversial in mainstream science. However, the strength of this approach is that the theory fits extremely well with the traditional metaphysics of the *Yijing* and, taken as a whole, seems to provide a coherent framework, expressed in the vernacular of science, which at least offers the potential of an explanation for divination.

Conclusion

Adler (1990 p15) provides the following summary of Zhu Xi's view of the function of the *Yi*:

> *The hexagrams of the I represent the yin-yang fluctuations and transformations of the Way. The I as a symbol system "discloses things" by representing natural pattern in graphic form, making it easier to comprehend.*

Bohm, speaking of the mathematicization of the ideas of implicate and explicate orders, says (1980 p205) that "an algebra contains key features which are similar to the key features of structures built on implicate orders." Boolean algebra is one of the most basic forms of algebra and, as I have shown extensively elsewhere, Boolean algebra can be fruitfully applied to the symbols of the *Yijing*. As such, I suggest that the *Yijing* forms a powerful notational system for exploring the manner in which the implicate order unfolds into the explicate, for exploring the manifestation of consciousness within pattern and matter. However, this is only part of the picture. Adler goes on to say:

> *But the Way, to be actualized, must also be internalized by the human will and put into effect in human affairs. Moral decisions must be made, based on an integrated understanding of the self and the world. Self-doubt is inevitable at this point, and the I provides a method by which doubts can be settled and intentions trained to issue spontaneously in a proper direction. Divination is this method.*

We have seen that a coherent picture of divination can be expressed in terms of a physics that directly parallels the metaphysics of the *Yi*. That the role of human consciousness is significant in this picture accords with Adler's characterization above. Further, the view of reality that this physics emphasises is a seamless, continuous field where human beings are deeply integrated into the basic fabric of reality through their consciousness. Once such a view is taken seriously, the only rational choice of action is an investigation into the causes of our perception of fragmentation and a

resulting movement towards realizing the true wholeness of reality. This is the essence of self-cultivation, as espoused by the great proponents of the *Yi*.

Zhu Xi, cited by Adler (1990 p8) says that we should "approach the old teaching in order to bring out new views." The work in this paper is presented in this spirit, and seeks to express the continuity of idea between the ancient sages of Change and contemporary thought. Bohm (1980 p31) is in agreement with the importance of this approach:

> *So what we have to do with regard to the great wisdom from the whole of the past, both in the East and in the West, is to assimilate it and to go on to new and original perception relevant to our present condition of life.*

I contend that the *Yijing*, both as a source of abstract philosophical study and as a practical tool for divination, provides an excellent method for generating "original perception relevant to our present condition".

References

Adler, Joseph A. 1990.
"Chu Hsi and Divination" in *Sung Dynasty Uses of the I Ching*, chs 6 and 7. Published by Princetown University Press, Princetown. Downloaded from www2.kenyon.edu/Depts/Religion/Fac/Adler/Writings/Divination.htm

———. 2009.
"On Translating *Taiji*". In *Zhu Xi Now*, edited by David Jones and He Jinli, forthcoming from Albany, SUNY Press. Downloaded from www2.kenyon.edu/Depts/Religion/Fac/Adler/Writings/Translating Taiji.pdf

Anathaswamy, Anil. 2010.
"Let Your Body Do the Thinking". In *New Scientist*, Volume 205, Number 2753, 27 March 2010, pp8-9. Published by Reed Business Information Ltd, England.

Barrow, John D. 2001.
The Book of Nothing. Published by Vintage Books, London.

Birdwhistell, Anne D. 1989.
Transition to Neo-Confucianism: Shao Yung on Knowledge and Symbols of Reality. Published by Stanford University Press, Stanford California.

Bohm, D. 1980.
Wholeness and the Implicate Order. Published by Routledge and Kegan Paul, Abingdon and New York.

———. 1992.
Thought as a System. Published by Routledge, Abindgon and New York.

Chaitin, G. J. 1975
"Randomness and Mathematical Proof" in *Scientific American*, Volume 232, Number 5, May 1975. pp 47-52. This is also available on the web at http://www.cs.auckland.ac.nz/CDMTCS/chaitin/sciamer.html

Combs, Alan and Mark Holland. 2001.
Synchronicity: through the eyes of science, myth, and the trickster, Published by Marlowe & Company, New York.

Ding, Zijiang. 2005.
"The Numerical Mysticism of Shao Yong and Pythagoras" in *Journal of Chinese Philosophy*, Volume 34, Number 4, pp615–632. Published by Blackwell Publishing in December 2005.
de Fancourt, William. 1997.
Warp and Weft. In Search of the I-Ching. Capall Bann Publishing, Freshfields.
Graham, Loren and Jean-Michel Kantor. 2009.
Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity. Published by The Belknap Press of Harvard University Press, Cambridge, Massachusetts.
Hamming, R.W. 1980.
"The Unreasonable Effectiveness of Mathematics". Originally published in *The American Mathematical Monthly*, Volume 87, Number 2, February 1980. Downloaded from http://www-lmmb.ncifcrf.gov/~toms/Hamming.unreasonable.html
Huang, Wen-Shan. 1984.
Fundamentals of Tai Chi Ch'uan. 5th Edition, published by South Sky Book Co, Hong Kong.
Jung, Carl G. 1955.
Synchronicity: An Acausal Connecting Principle. Republished by Ark Paperbacks in 1985.
———. 1960.
On the Nature of the Psyche. Published by Routledge, London in 2001.
Lloyd, Seth. 2006.
Programming the Universe: A Quantum Computer Scientist Takes on the Cosmos. Published by Jonathan Cape, London.
Long, Jeffery G. 1999.
"How Could the Notation be the Limitation?". Published in *Semiotica: Journal of the International Association for Semiotic Studies*, Volume 125, Issue 1-3, Pages 21–32, by Walter de Gruyter GmBH & Co.
———. 2005.
"Notation Systems and Cognitive Evolution". Presented at the 2005 Annual Conference of the American Society for Cybernetics. Downloaded from www.gwu.edu/~rpsol/ASC/Paper/long.doc
Lynn, Richard John. 1994.
The Classic of Changes: A New Translation of the I Ching as Interpreted by Wang Bi. Translated by Richard John Lynn. Published by Columbia University Press, New York.
McFarlane, Thomas J. 1995.
"The Spiritual Function of Mathematics and the Philosophy of Franklin Merrell-Wolff". Downloaded from http://www.integralscience.org/sacredscience/SS_spiritual.html
Miller, George A. 1956.
"The Magical Number Seven, Plus or Minus Two: Some Limits on our Capacity for Processing Information". Originally published in *Psychological Review*, Volume 101, Number 2, pages 343-352. Downloaded from http://www.psych.utoronto.ca/users/peterson/psy430s2001/Miller GA Magical Seven Psych Review 1955.pdf
Nielsen, Bent. 2003.
A Companion to Yi Jing Numerology and Cosmology: Chinese Studies of Images and Numbers from Han (202 BCE – 220 CE) to Song (9 60 – 1279 CE). Published by Routledge Curzon, London.

Penrose, Roger. 2005.
The Road to Reality: A Complete Guide to the Laws of the Universe. Published by Vintage Books, London.
Russell, Bertrand. 1917.
Mysticism and Logic, Including a Free Man's Worship. Republished by Routledge. London 2004.
Ryan, James A. 1996.
"Leibniz' Binary System and Shao Yong's Yijing" Published in *Philosophy East & West*, Volume 46, Issue 1, 1996.
Schöter, Andreas. 1998.
"Boolean Algebra and the Yijing" in *The Oracle: the Journal of Yijing Studies*, Volume 2, Number 7, Summer 1998, pp19–34.
———. 1999.
"Correctness and Correspondence" in *The Oracle: the Journal of Yijing Studies*, Volume 2, Number 8, February 1999, pp27–35.
———. 2004a.
"Great Polar Fist and the Book of Change" in *Tai Chi Chuan and Internal Arts, the Journal of the Tai Chi Union for Great Britain*, Number 18, Spring 2004, pp14–17.
———. 2004b.
"The Nature of Divination" in *Counselling in Scotland*, Spring/Summer 2004, pp6–8. The Journal of COSCA, Stirling.
———. 2004c.
"Flowers and Steps in the Boolean Lattice of Hexagrams" in *Journal of Chinese Philosophy*, Volume 31, Number 4, pp489–504. Published by Blackwell Publishing in December 2004.
———. 2005.
"The Yijing as a Symbolic Language for Abstraction" in *Proceedings of the 2nd International Conference on I-Ching (Yijing) Studies and Contemporary Civilization*, pp291–305.
———. 2008.
"Bipolar Change" in *Journal of Chinese Philosophy*, Volume 35, Number 2, pp297–317. Published by Blackwell Publishing in June 2008.
Seife, Charles. 2006.
Decoding the Universe: How the New Science of Information is Explaining Everything in the Cosmos from Our Brains to Black Holes. Published by The Penguin Group, New York.
Shapiro, Stewart. 2000.
Thinking about Mathematics: the Philosophy of Mathematics. Published by Oxford University Press, Oxford.
Sheldrake, Rupert. 2009.
A New Science of Life. Third edition. Published by Icon Books Ltd., London.
Suler, John R. 1993.
Contemporary Psychoanalysis and Eastern Thought. Published by State University of New York Press, Albany.
Tegmark, M. 2007.
"Reality by Numbers: What is the Universe Really Made of?". In *New Scientist*, Volume 195, Number 2621, 15 September 2007, pp38-41. Published by Reed Business Information Ltd, England.

Wigner, E. 1960.
"The Unreasonable Effectiveness of Mathematics in the Natural Sciences". Originally published in *Communications in Pure and Applied Mathematics*, Volume 13, Number 1, February 1960, by John Wiley and Sons, Inc., New York. Downloaded from http://www.ipod.org.uk/reality/reality-wigner.pdf
Wu, Jing-Nuan. 1991.
Yi Jing. Published by The Taoist Center, Washington D.C.
Yang, Jwing-Ming. 1996.
Tai Chi Theory and Martial Power. Published by YMAA Publication Center, Massachusetts.