Correctness and Correspondence*

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1. Introduction

Much of the commentary for the individual lines in various versions of the Yi Jing are concerned with the issues of *correctness*, *holding together* and *correspondence*. For example, the work of Cheng Yi applies the theories of Inner Design to develop a detailed commentary of the Yi that reflects the application of these concepts. Cheng Yi's work is translated as *The Tao of Organization* by Cleary [Cle88] and I will have recourse to make extensive reference to this commentary throughout this paper. I shall begin, in Section 2 by describing the traditional notions of correctness and correspondence, as presented by Cleary and Cheng Yi in [Cle88]. This includes a description of the roles of the individual lines, as this helps to provide a context for understanding the significance of correctness and correspondence when used in interpreting the structures of the Yi.

Then, in Section 3, I shall present a method for representing correctness and correspondence, using the algebraic tools that I discussed in my earlier paper "Boolean Algebra and the Yi Jing" [Sch98]. I shall use those techniques to develop formal definitions for these properties and then use those definitions to prove some interesting characteristics. In particular, in Section 3.5, I shall show that correctness and correspondence are interconnected properties, rather than two unconnected notions. As an interesting aside I shall also show how correspondence can be used to provide a method for classifying the hexagrams into eight classes. I shall not discuss the property of holding together in this essay.

The reader not familiar with my earlier paper is referred to Section 5 where I present a brief summary of the notation and formal techniques used throughout this work. These are presented in more detail in my earlier paper [Sch98].

Because of the extensive use of Cleary's translation in this paper I have decided to adopt the names of the hexagrams used there, rather than the more familiar translations offered by Wilhelm. To make things easier on the reader I therefore also include the King Wen sequence number and a linear binary representation of the gua where appropriate.[1]

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2. The Traditional Perspective

In the following sections I present a brief overview of the traditional perspective on the relationships of correctness and correspondence. Given that both of the notions I shall be investigating are concerned with the properties of, and relationships between, the lines, it is informative to begin with a brief description of their individual characteristics.

2.1 The Individual Lines

The following attribution of roles gives one particular analysis of the qualities of the individual lines:

1st line: unskilled workers.
2nd line: skilled workers.
3rd line: middle management.
4th line: personal assistants.
5th line: senior management.
6th line: external consultants.

This system of attributions is a summary of the Inner Design system used by Cheng Yi as it would be applied in an organizational context. One way of viewing correctness and correspondence then, is as a system for considering the attribution of yin and yang to these various organizational roles and for analyzing the relationships between the roles once the attributions have been made.

Cleary describes this particular system in more detail in [Cle88, pp*xii-xiii* & *xvii*] and describes a number different line systems (including the one presented above) from Taoist, Buddhist and Confucian perspectives in [Cle89, pp21-29].

2.2 Correctness

The notion of correctness relates the nature of a line (whether it is a yin or a yang line) to its place in the overall structure of the hexagram. The idea is that certain places in a hexagram are yang in nature and therefore it is usually more appropriate (correct) for a yang line to take that place; similarly, certain places are taken to be yin in nature and it is generally correct for a yin line to take that place.

In the eighth wing of the Yi, the division of the text that Wilhelm refers to as “The Discussion of the Trigrams” (the *Shuo Gua*), we have the statement that “to heaven they assigned the number three and to earth the number two; from these they computed the other numbers” [Wil83, p262]. This is spelt out step by step in Chapter IX, verse 1 of the *Dazhuan* where the first ten numbers are assigned to heaven and earth: “Heaven is one, earth is two, heaven is three, earth is four, heaven is five, earth is six…” and so forth [Wil83, p308]. Following on from this we have that “The places are divided into the dark and light. The yielding and the firm occupy these by turns." [Wil83, p264]. So, as might be expected from the general numerology expressed here, odd numbered places in a hexagram are yang in nature and even numbered places are yin in nature.[2]

According to this system then, hexagram 63 *After Completion* **101010** has every line correct: the first, third and fifth lines are yang whilst the second, fourth and sixth lines are yin. As Wilhelm notes, this is the only hexagram in which all the lines stand in their proper places [Wil83, p709]. In this context Cleary's translation of the name of this hexagram as *Settled* is very apt [Cle88, p209].

But by the same reasoning, in hexagram 64 *Before Completion* **010101** every line is incorrect: the first, third and fifth lines are yin when they should be yang, whilst the second, fourth and sixth lines are yang when they should be yin. Again, Cleary's translation of the name of this gua as *Unsettled* is quite appropriate when we consider the correctness of the lines.

Clearly, other hexagrams will have various mixes of correct and incorrect lines. Some examples from Cheng Yi's commentary will bring out these points. Firstly, we shall consider some incorrect lines.

For the 3rd yin in hexagram 10 *Treading* **110111** we have yin in a yang position. Cheng Yi's commentary for this line is "This is one who wants to be strong but who is basically weak and cannot be firm in action." Conversely, for the 4th yang in hexagram 35 *Advance* **000101** we have yang in yin place being inauspicious; Cheng Yi says "Yang in the fourth line is out of place. Remaining where it is out of place, it represents those who occupy a position out of greed."

As an example of correctness we can consider the 4th yin in hexagram 37 *People in the Home* **101011**. This is yin in a yin place and is taken as auspicious. Cheng Yi says "To be in the proper place means to live securely, at peace."

Whilst these examples show the typical application of correctness, Wilhelm [Wil83, p361] notes that being incorrect may not always be a disadvantage. Again, the following examples from Cheng Yi's commentary make this clear.

We find that for the 2nd line in hexagram 34 *Great Power* **111100** there is yang in a yin place. Cheng Yi says "the second line has yang strength dealing with a time of great power, nevertheless it remains flexible". So the nature of the line benefits from the spirit of its place. Similarly, for the 1st yin in hexagram 40 *Solution* **010100** Cheng Yi says "it has flexibility in a position for strength, responding to yang with yin. This means being flexible yet able to be firm."

Equally, correctness may not always be an advantage. Cheng Yi's commentary contains some examples of these kinds of cases. Considering hexagram 34 *Great Power* again, in the 3rd yang we have a correct line, but it is too yang. The comment says “The third yang has strength in a yang position, dealing with power... This is like when people esteem power and use it on whatever they confront, inevitably being frustrated.” Similarly, in the 4th yin of hexagram 36 *Damage to Illumination* **101000** we have a yin line in a yin place being too yin. Cheng Yi says that “This represents dishonest petty people in high positions following the leadership obediently, being weak and devious.”

Thus, although the property of correctness gives us the basic framework for interpreting the nature of the lines, the actual determination of whether the result is auspicious or not depends very much on the overall context of the gua. The algebraic analysis given in Section 3.2 addresses the formal properties of correctness and not its contextual interpretation.

2.3 Correspondence

The notion of correspondence relates pairs of lines within a hexagram. However, correspondence between lines cannot be taken as a given - it usually only occurs when a line in a certain position has a corresponding complementary line in a related position.

Which places give rise to correspondences? If we take the upper and lower trigrams of a hexagram and match them line for line, then the first lines of each trigram may correspond; similarly, the second lines of each trigram and the top lines of each trigram are also potential correspondents. Specifically, when the first and fourth lines are complements, then they correspond; similarly the second and fifth lines may correspond and the third and sixth lines also. Figure 1 shows these connections graphically.

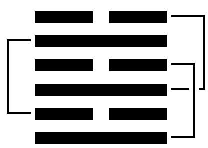

Figure 1: Correspondence

Richard Smith translates the Chinese term for this relationship as “correctly resonates” [Smi98, p41]; this phrase captures the intent behind the idea of correspondence.[3] Cheng Yi says in the commentary to the third yin in *Reduction* (hexagram 41, **110001**) “when each pair of lines is complementary, then the aim is unified; for each one this is getting a companion” [Clea88, p134]. Thus, if we consider this remark and the general nature of Cheng Yi's other comments on the subject, we can see correspondence as defining a relationship of mutual attraction and helping between pairs of lines.

It is interesting to consider hexagram 64, *Unsettled*, again in the light of both correctness *and* correspondence. Wilhelm says of this hexagram that "outwardly viewed, none of the lines appears in its proper place; but they are all in relationship to one another" [Wil83, p714]. What is meant by this is that although, as previously noted, every line is incorrect, each line stands in correspondence with its pair from the other trigram.

Some examples from Cheng Yi's commentary will help clarify the application of this relationship. Consider the 1st yin in hexagram 6 *Contention* **`010111`**; this line has a correspondent in the 4th yang. This correspondence gives the otherwise incorrect line a positive angle. Cheng Yi says "It is because there is corresponding assistance from a higher level that people in this position are able to refrain from persisting forever in an affair, and are lucky to get by with a little criticism." The comment for the corresponding 4th yang says that "the first [line] is in the right cooperative relationship and is obedient, so it is not a party to contention." In contrast, compare the 2nd and 5th yangs; their positions should correspond, but they are not complements. Cheng Yi says that "...they are both strong, so they contend."

For the 2nd yin in hexagram 17 *Following* **`100110`** the situation is quite complex. It corresponds with the 5th yang but is drawn to the nearer 1st yang.[4] Cheng Yi says "if you get involved with the small child you will lose the adult. The first yang below is the small child; the fifth, the true correspondent above, is the adult. If the second gets concerned with the first, it will lose its true correspondent, the fifth yang."

In contrast we can consider the 1st yang in hexagram 13 *Association with Others* **`101111`**. This line has no correspondent, but Cheng Yi's comment says that "this signifies absence of personal bias." Thus, as with correctness, although the abstract definition of correspondence provides the framework, the actual interpretation based on the relationship depends on the context of the situation. The most blatant example of this is in the 1st and 4th yangs in hexagram 38 *Disharmony* **`110101`**. For Cheng Yi these lines correspond! He says of the situation "only the first and fourth, though they are not complementary, have the same qualities and associate together, so they harmonize."

Again, the algebraic analysis of correspondence in Section 3.3 is concerned with the formal aspects of the relationship, rather than its contextually determined interpretation.

3. The Algebraic Perspective

This section builds on the formal approach that I began to develop in my earlier paper [Sch98]. In that work I used mathematical techniques to analyse two relationships put forward by Cleary (specifically the Structural Complement and Primal Correlate discussed in [Cle89, pp29–31]). In this section I apply the same tools to explore an algebraic perspective on the two traditional relationships discussed above.

Technically, correctness can be seen as a relationship between an individual line and its place in the gua, whilst correspondence is a relationship between two lines according to their relative positions. Paralleling the representation of the context of a change by a hexagram in [Sch98, §3.2.3], the formal representations of correctness and correspondence will be in terms of gua. We shall see that correctness for a hexagram will be represented by another hexagram, whilst correspondence for a hexagram will be represented by a trigram.

3.1 Some Auxiliary Notation

In order to offer a formal approach to these notions it is necessary to make some notation explicit that it was possible, for the sake of simplicity, to leave implicit in my earlier work. Specifically, I shall need to introduce a notation for talking about the gua as an ordered sequence of lines. Informally, in the text, I shall continue to write a hexagram, for example, *The Well*, as an undifferentiated linear figure **`011010`**. However, the reader should understand this as shorthand for the more explicit form `[`**`0,1,1,0,1,0`**`]` where each line is properly distinguished from the others in an ordered linear structure.

This is necessary because, in some parts of the following discussion, I need to break a gua down into smaller units. Where it causes no confusion I will continue to use the original, simpler form of notation.

3.2 Correctness

I wish to propose the following formal representation of the relation of correctness. The correctness of the lines in a hexagram `G` can be represented by another hexagram, its *correctness* hexagram `C`. In the correctness hexagram a line is yang if the equivalent line in the original hexagram is correct; similarly, a line is yin in the correctness hexagram if the equivalent line in the original hexagram is not correct.

Consider, as given above, that every line in **`101010`** is correct and conversely, that every line in **`010101`** is incorrect. This leads to the following definition:

Definition 1: Correctness

> The correctness hexagram `C` of a hexagram `G` is
> `C = G^`**`010101`**

One way to view the correctness of a hexagram is to see it as the difference between that hexagram and absolute incorrectness. This is what Definition 1 captures in a formal statement. If we apply this definition to the examples given above we get the following results. Firstly, consider the hexagram *Settled* **101010**. From Definition 1 we get:

```
G = 101010
C = 101010^010101
  = 111111
```

This is the required result; every line in *Settled* is correct so its correctness hexagram should be composed of all yang lines. Now consider *Unsettled* **010101**. Again, applying the definition gives:

```
G = 010101
C = 010101^010101
  = 000000
```

Which is the result that the initial analysis suggested; all the lines in *Unsettled* are incorrect so its correctness hexagram should be all yin lines. Finally, consider the example of hexagram 56 *Travel* **001101**. The definition will give

```
G = 001101
C = 001101^010101
  = 011000
```

Again, this gives us the desired result. The only correct lines in *Travel* are the 2nd yin and the 3rd yang and both of these lines are yang in the correctness hexagram with all other lines being yin.

Thus, we have a formal relation that gives us a representation of correctness in the following terms. The correctness of a hexagram is defined as another hexagram: for each correct line in the initial hexagram we have a yang line in the result, and for each incorrect line we have a yin line.

I shall now present some properties of correctness that can be verified using the formal tools of Boolean algebra.

Theorem 1: Unique Correctness

Each hexagram has a unique correctness hexagram.

The proof of this is simple. It is a theorem of general Boolean algebra that if A≠B then A^X≠B^X for any values of A, B and X. For our purposes X is fixed as **010101** in Definition 1, and A and B are allowed to vary freely. Thus, we can see that if A and B are distinct hexagrams, then their correctness hexagrams will also be different.

The next theorem relates the correctness of a hexagram with the correctness of its complement.

Theorem 2: Complementary Correctness

> If the correctness hexagram of `G` is given by `C`, then the correctness hexagram of `~G` will be `~C`.

That is, in English, if a line is correct in `G` then it will be incorrect in `~G`.

The proof of this is simple. As a general rule of Boolean algebra we have:

```
(~X)^Y = ~(X^Y)
```

Which, in words, says that the difference between the complement of `X` and `Y` is the same as the complement of the difference between `X` and `Y`. In the definition of correctness `Y` is fixed as **`010101`** and `X` is the gua we are interested in finding the correctness of. Theorem 2 now follows directly:

if `C = G^`**`010101`**, then for `~G` we have

```
    ~G^010101
  = ~(G^010101)
  = ~C
```

Theorem 3 involves the correctness of a hexagram composed of a single trigram doubled.

Theorem 3: Correctness for Doubled Trigrams

> If `G = [a, b, c, a, b, c]`, then its correctness hexagram `C` will have the form `[d, e, f, ~d, ~e, ~f]`.

In English this means: if a hexagram has the same trigram repeated, then its correctness hexagram will have complementary trigrams. The implication of this theorem for correctness is that in a hexagram composed from a repeated trigram, at most half of the lines can be correct. Further, preempting some of the results from Section 3.5, if a line is correct in the lower trigram, then its corresponding line in the upper trigram will be incorrect and *vice versa*.

The proof of this theorem is straightforward. If a hexagram has the same trigram repeated it will have a line configuration in the form

```
G = [a, b, c, a, b, c]
```

Now, the correctness difference hexagram **`010101`** has the form

```
[d, e, f, ~d, ~e, ~f]
```

Thus, from Definition 1, the correctness hexagram `C = G^`**`010101`** will be:

```
[a, b, c, a, b, c] ^ [d, e, f, ~d, ~e, ~f]
```

Taking this expression line by line, this expands to:

```
[a^d, b^e, c^f, a^~d, b^~e, c^~f]
```

And then, using the same rule of Boolean algebra that we used in the proof of Theorem 2, we can transform this to:

```
[a^d, b^e, c^f, ~(a^d), ~(b^e), ~(c^f)]
```

Which has the required form for the conclusion of the proof.

The next theorem presents the correctness property for hexagrams composed from a pair of complementary trigrams.

Theorem 4: Correctness for Paired Complements

If `G = [a, b, c, ~a, ~b, ~c]`, then its correctness hexagram `C` will have the form `[d, e, f, d, e, f]`.

Or, in English, if a hexagram has complementary trigrams, then its correctness hexagram will have the same trigram repeated. This is the converse of Theorem 3: for a hexagram composed of complementary trigrams, if a line in the lower trigram is correct, then its corresponding line in the upper trigram will also be correct.

The proof of this theorem follows the same pattern as the proof for Theorem 3 and I omit its detailed presentation here.

3.3 Correspondence

Let us now turn to correspondence. This notion can be seen as a comparison of the lines from the two primary trigrams of a hexagram. Consider the following definition:

Definition 2: Correspondence

For hexagram `G = [a, b, c, d, e, f]`,
the correspondence trigram `R = [a^d, b^e, c^f]`

This definition represents the correspondence of a hexagram as a trigram. This is strictly correct, correspondence relates pairs of lines in a hexagram and there are three pairs of lines. In this case a yang line in the first position in the correspondence trigram indicates that the first and fourth lines of the hexagram correspond, a yang line in the second place in the trigram indicates correspondence between the second and fifth lines of the hexagram, and a yang line in the top place in the trigram indicates correspondence between the third and sixth lines of the hexagram. The following examples will show how this definition works.

Consider hexagram 63, *Settled*, **101010**. In addition to being completely correct this hexagram also has every line standing in correspondence. If we apply Definition 2 to **101010** we get

```
R = [1^0, 0^1, 1^0]
  = [1, 1, 1]
```

Thus we see that the correspondence for *Settled* is **111** indicating that every pair of lines is in correspondence. Similarly consider hexagram 64, *Unsettled*, **010101**. As previously noted, although every line is incorrect, every line is actually in proper correspondence with its pair. Again, applying the definition we get:

```
R = [0^1, 1^0, 0^1]
  = [1, 1, 1]
```

So, the correspondence for *Unsettled* is also **111** indicating that every pair of lines is in correspondence. Now let us consider hexagram 56, *Travel* **001101**. In this hexagram only the first line has a correspondent: the first yin and the fourth yang are complements and in the correct relative position. If we apply Definition 2 we get the following result:

```
R = [0^1, 0^0, 1^1]
  = [1, 0, 0]
```

As for correctness, there are some interesting theorems that can be proven for the property of correspondence. Theorem 5 for correspondence parallels Theorem 1 for correctness.

Theorem 5: Correspondence Classes

Every hexagram falls into one of eight correspondence classes.

This theorem follows directly from Definition 2. If the correspondence of a hexagram is represented by a trigram then, as there are eight trigrams, every hexagram must fall into a correspondence group matching one of the trigrams. The implications of this idea are explored a little in Section 3.4 below.

The next theorem for correspondence parallels Theorem 3 for correctness.

Theorem 6: Correspondence for Doubled Trigrams

For a hexagram `G = [a, b, c, a, b, c]`, `R = [0, 0, 0]`.

In English, this simply says that any hexagram composed of a trigram doubled has no corresponding lines. The proof of this is direct. If

```
G = [a, b, c, a, b, c]
```

then from Definition 2 we know that

```
R = [a^a, b^b, c^c]
```

And as it is a rule of Boolean algebra that `X^X` = **`0`** for any `X`, we can see that

```
R = [0, 0, 0]
```

Theorem 7 for correspondence parallels Theorem 4 for correctness. It is the converse of Theorem 6 above.

Theorem 7: Correspondence for Complementary Trigrams

For a hexagram `G = [a, b, c, ~a, ~b, ~c]`, `R = [`**`1`**`, `**`1`**`, `**`1`**`]`.

This says that any hexagram composed of a pair of complementary trigrams has every pair of lines in correspondence. Again, the proof is direct. If

```
G = [a, b, c, ~a, ~b, ~c]
```

Then from the definition we know that

```
R = [a^~a, b^~b, c^~c]
```

And as it is a rule of Boolean algebra that `X^~X = `**`1`** for any `X`, we can see that

```
R = [1, 1, 1]
```

3.4 Correspondence Classes

Before considering the relationship between the two properties it is interesting to briefly look at an intriguing result of Theorem 5. This theorem identified the fact that every hexagram falls into one of eight possible patterns of correspondence, where those patterns of correspondence can be related to the eight trigrams. Table 1 shows this classification. Now we can see that, in addition to each hexagram falling into one of the eight classes, each of the classes contains exactly eight hexagrams. Thus, correspondence gives us a new way of dividing the hexagrams of the Yi into families.

The left-most column, headed by the trigram **`000`**, contains those hexagrams which have no corresponding lines. As we would expect from Theorem 6 these are those hexagrams that are composed of a single trigram doubled. Similarly, the right-most column, headed by **111**, contains that hexagrams for which every pair of lines is in correspondence. Again, as Theorem 7 stipulates, these are those hexagrams composed from complementary trigrams.

000	100	010	110	001	101	011	111
000000	000100	000010	000110	000001	000101	000011	000111
001001	001101	001011	001111	001000	001100	001010	001110
010010	010110	010000	010100	010011	010111	010001	010101
011011	011111	011001	011101	011010	011110	011000	011100
100100	100000	100110	100010	100101	100001	100111	100011
101101	101001	101111	101011	101100	101000	101110	101010
110110	110010	110100	110000	110111	110011	110101	110001
111111	111011	111101	111001	111110	111010	111100	111000

Table 1: The Correspondence Classes

The remaining columns contain the hexagrams with intermediate degrees of correspondence. For example, the fourth column, headed by the trigram *The Joyous* **110**, contains those hexagrams in which the first and fourth lines correspond, the second and fifth lines correspond, but the third and sixth lines do not. An example of this is hexagram 40 *Solution*, **010100**.

The possible importance of this method of classifying the gua remains to be explored.

3.5 Relating Correctness and Correspondence

Correctness and correspondence are not independent concepts. In this section I shall take the definitions offered in Sections 3.2 and 3.3 and show how these can be used to make the interconnections between the two notions explicit. These interconnections are spelt out formally by means of a number of statements which are gathered together as Theorem 8.

I shall provide a formal proof of Theorem 8a and Theorem 8c. The proofs for the remaining parts of the theorem follow the same pattern and are left as an exercise for the reader.

Theorem 8: Correctness and Correspondence.

a) If a line is correct and has a correspondent, then that line is correct too.
b) If a line is incorrect and has a correspondent, then that line is incorrect too.
c) If a line is correct and has no correspondent, then the relevant line is incorrect.
d) If a line is incorrect and has no correspondent, then the relevant line is correct.

The proof of Theorem 8a proceeds as follows. We are interested in the situation where a line is correct and has a correspondent, we then show that the corresponding line is also correct. Suppose the hexagram under consideration is:

`G = [a, b, c, d, e, f]`

Then we know from Definition 1 that the correctness hexagram will be:

`C = [a^0, b^1, c^0, d^1, e^0, f^1]`

We further know from Definition 2 that the correspondence trigram will be:

`R = [a^d, b^e, c^f]`

Now suppose that line 1 is correct and it has a correspondent. Then we can determine from the preceeding expressions that:

`a^0` = **`1`** (line 1 is correct) and `a^d` = **`1`** (lines 1 and 4 correspond)

What we now need to show is that line 4 is correct. This would be represented as

`d^1` = **`1`**

Now, given that `a^0 = 1`, looking at the table for the difference operator (see Table 6 in Section 5) we can conclude that `a = 1`. Now, we can substitute this value into the expression `a^d = 1` to give `1^d = 1` which is what we needed to prove. The other two pairs of lines have the same pattern of proof. Thus, we have shown the property holds for each related pairs of lines in a hexagram, therefore Theorem 8a is true.

The proof of Theorem 8c is similar. We are interested in the situation when a line is correct and has no correspondent, we then show that the relevant line is incorrect. We can use the expressions for `G`, `C` and `R` from above. Now, suppose that line 1 is correct but has no correspondent. Then we can determine that:

`a^0` = **`1`** (line 1 is correct) and `a^d` = **`0`** (lines 1 and 4 do not correspond)

Now we need to show that line 4 is incorrect. This would be represented as

`d^1 = 0`

Now, given that `a^0 = 1` as before we know that `a = 1`. Now we can substitute this value for `a` into `a^d = 0` to give `1^d = 0` which is what we needed to prove. Again, the other two pairs of lines follow the same pattern of proof. Thus, given that the property holds for each of the related pairs of lines in a hexagram, we have shown that the theorem holds in general.

The proofs for Theorem 8b and Theorem 8d are essentially the same and the details are omitted here.

4. Conclusions

In “Boolean Algebra and the Yi Jing” [Sch98] I introduced a mathematical analysis of some of the structural aspects of the gua. In that paper I then apply the formal techniques to provide a description of a number of phenomena. In particular I describe how Cleary’s relations of the Structural Complement and the Primal Correlate [Cle89, pp29–31] can be given a precise formal description. I also show how the relationship that holds between the initial gua in a reading and the gua resulting from its changing lines can be given a mathematical analysis within the same framework.

In this paper I take the same formal tools from [Sch98] and apply them to provide a detailed analysis of the traditional notions of correspondence and correctness. In particular I have shown that correctness and correspondence are not independent notions. Rather they fit together in a precise and well defined way as is brought out by the theorems in Section 3.5. Further, as a side effect of Theorem 5, we find a new way of classifying the hexagrams into eight families, although the significance of this result needs to be explored in more depth.

The tools have proven capable of providing a detailed description of notions introduced by Cleary, of providing an analysis of the process of change within a reading and, in this paper, of analyzing two traditional notions. I believe that the broad success of the formal analyses that I have been able to provide using these tools is indicative of their general applicability to the structures in the Yi.

5. Technical Summary

The formal techniques used in this paper rely on Boolean algebra. In this section I shall present a brief summary of the notation used. For a fuller description of the mathematics, the reader is referred to my earlier paper [Sch98], especially Section 2, from which this summary draws heavily. Table 2 below shows the logical operators that we shall have cause to use in this essay.

Logical name	**Interpretation**	**Notation**
not	complement	`~x`
or	union	`x|y`
and	intersection	`x&y`
xor	difference	`x^y`

Table 2: The Combinatory Operations

The first of these is a *unary* operation in that it applies to a single gua to generate another. This is defined by Table 2 from [Sch98, §2.1.1] repeated below as Table 3. The *complement* of a yin line is a yang line and the complement of a yang line is a yin line.

x	~x
0	**1**
1	**0**

Table 3: *complement* operation

That is, for individual lines, `~0 = 1` and `~1 = 0.` The effect on structures is the same, taken line by line. For example, consider Water, it is transformed into Fire: ~**010** = **101**. Similarly, the Arousing is transformed into the Gentle: ~**100** = **011**.

The remaining operators are binary: that is, they take two gua and generate a third. The first of these is *union*. Where `x|y = z`, then `z` is the result of taking the union of the yang energies in `x` with the yang energies in `y`. This is summarized in the following table:

x	y	x|y
0	**0**	**0**
0	**1**	**1**
1	**0**	**1**
1	**1**	**1**

Table 4: The *or* operation

Again, the extension to structures is done line by line. For example, in trigrams, the result of combining the Arousing with Stillness gives Fire: **100|001** = **101**.

The next operator is *intersection*. Where `x&y = z`, then `z` is the result of taking the intersection of the yang energy in `x` with the yang energy in `y`. Again, it is best to represent this in a tabular form:

x	y	x&y
0	**0**	**0**
0	**1**	**0**
1	**0**	**0**
1	**1**	**1**

Table 5: The *and* operation

As with the other operations, this is ready extended to structures by taking them line by line. For example, combining the energies of the Joyous with the energies of Fire through the *and* operation gives the Arousing: **110**&**101** = **100**.

The final Boolean operator that we shall use is the *difference* operator. That is, where `x^y = z`, `z` is the difference between `x` and `y`. As before, this is best represented in a tabular format:

x	y	x^y
0	**0**	**0**
0	**1**	**1**
1	**0**	**1**
1	**1**	**0**

Table 6: The *xor* operation

This is perhaps the most interesting of the structural operations. Considering its action on trigrams, the Abyss would represent the difference between the Arousing and the Joyous: **100^110** = **010**.

The final two symbols that I use are = for *equality* and ≠ for *inequality*. For example, A=B simply means A and B are equal, whilst A≠B means that A and B are not equal.

6. Notes

[1] The reader is reminded that it is this author's preference to write the linear representation of gua as left-to-right for bottom-to-top. Thus the trigram *Zhen* is written as **100**. There is no real consensus around this issue and the decision is essentially arbitrary. Cleary [Cle89] and Hacker [Hac93] both opt for the left-to-right is bottom-to-top representation, but others prefer the left-to-right is top-to-bottom representation. I use the former representation as many of the formal definitions in this and my earlier paper use recursive definitions which are naturally expressed left-to-right in a programming context.

[2] A more detailed description of the property of correctness is given by Wilhelm in Section 5 of "The Structure of the Hexagrams" [Wil83, pp360-361].

[3] Wilhelm discusses this relationship in detail in Section 6 of "The Structure of the Hexagrams" [Wil83, pp361-362].

[4] In fact, this is the relationship of *holding together*. Two lines may hold together when they are adjacent in a hexagram. This relationship does not concern me in this paper, but the interested reader is referred again to Wilhelm's "The Structure of the Hexagrams" [Wil83, pp362–364].

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